MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 412: FLUID MECHANICS II

Date: 30th November, 2016 Time: 3.30 - 6.30 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Start each question on a new page.
- Indicate question numbers clearly at the top of each page.
- Observe further instructions on the answer booklet.

Question One: (30 marks)

A two dimensional source of strength q is placed at a distance h from an infinitely long wall. Body forces are absent and pressure at infinity and in the region behind the wall(i.e.)$x > 0$) is P_∞, the stagnation pressure

(a) Sketch the problem [3 marks]

(b) Show that the magnitude of the force, F, per unit length of the wall, in the $x-$direction, is given by .

$$F = \frac{-\rho q^2}{4\pi h},$$

where ρ is the density of the fluid [13 marks]

The velocity field of an incompressible flow is given by

$$\mathbf{v} = -2ay\mathbf{i} - 2ax\mathbf{j},$$

where a is a constant

(c) Show that the flow is irrotational [4 marks]

(d) Determine an expression for the velocity potential [5 marks]

(e) Determine an expression for the stream function. Given that $a > 0$, sketch the streamlines if the flow is confined in the region $x \geq 0$ [5 marks]

Question Two: (20marks)

Consider a velocity potential in cylindrical coordinates given by

$$\phi = \frac{\mu \cos\theta}{r}$$

(a) Write down an expression for the velocity v [3 marks]

(b) State the relationship between the components of velocity, the velocity potential and the stream function. Hence obtain an expression for the stream function. By changing to Cartesian coordinates, show that

$$x^2 + (y - \frac{1}{2k})^2 = \left(\frac{1}{2k}\right)^2,$$

where k is a constant. Hence sketch the streamlines [17marks]

Question Three: (20 marks)

(a) Show that if

$$\phi_1, \phi_2, \phi_3, \ldots, \phi_n$$

satisfy Laplaces equation, then the sum also satisfies Laplaces equation [3 marks]

(b) If the velocity components of a two-dimensional flow are

$$u(x,y) = \frac{k(x^2 - y^2)}{(x^2+y^2)^2}, \quad v(x,y) = \frac{2kxy}{(x^2+y^2)^2},$$

where k is a constant, show that this flow is incompressible[11marks]

(c) Define the following terms

(i) upstream velocity

(ii) stagnation plane

[6marks]

Question Four: (20 marks)

(a) A complex potential w can be expressed as follows

$$w = \phi + i\psi$$

where ϕ is the velocity potential and ψ is the stream function. Derive the relationship between ϕ and ψ [9 marks]

(b) A fixed cylinder is placed in a liquid which is moving steadily and irrotationally. If M is the moment about the origin, then neglecting external forces, show that

$$M = \text{real part of} - \frac{1}{2}\rho \oint_c z \left(\frac{dw}{dz}\right)^2 dz$$

where w is the complex potential, ρ density and the integral is taken around the contour of the cylinder [11marks]

Question Five: (20marks)

(a) Distinguish between

(i) inviscid and viscous flow

(ii) compressible and incompressible flow

[4marks]

(b) The complex potential of a flow is given by

$$w = az^2$$

where a is a constant. Determine the velocity potential and the stream function. Hence sketch the streamlines and the equipotential lines

[16marks]