MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 413: METHODS II

:te: 29th November, 2016 Time: 12.00 - 3.00 pm

ISTRUCTIONS:

Answer question ONE and any other TWO questions.

Observe further instructions on the answer booklet.

QUESTION ONE (Compulsory) **[30 Marks]**

(a) Given $\frac{\mathrm{d}}{\mathrm{d}x}[x^n J_n] = x^n J_{n-1}$, $\frac{\mathrm{d}}{\mathrm{d}x}[x^{-n} J_n] = -x^{-n} J_{n+1}$, and $J_{\frac{1}{2}}(x) = (\frac{2}{\pi x})^{\frac{1}{2}} \sin x$. Show that:

(i) $J_{-\frac{1}{2}}(x) = (\frac{2}{\pi x})^{\frac{1}{2}} \cos x$. [5 Marks]

Hence find

(ii) $J_{\frac{3}{2}}(x)$ [3 Marks]

and

(iii) $J_{-\frac{3}{2}}(x)$ [3 Marks]

(b) Show that $\dfrac{1-t^2}{(1-2xt+t^2)^{\frac{3}{2}}} = \sum_{n=0}^{\infty}(2n+1)P_n t^n$. [7 Marks]

(c) Use differentiation to solve:

$$y(x) = x - \int_0^x xz^2 y(z)\,\mathrm{d}z.$$

[7 Marks]

(d) Change the following differential equation into a Volterra integral equation:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \omega^2 y = 0$$

such that $y(0) = 0$ and $y'(0) = 1$. [5 Marks]

QUESTION TWO **[20 Marks]**

(a) Find the resolvent kernel of the integral equation:

$$u(x) = 1 + \lambda \int_0^1 xtu(t)\mathrm{d}t$$

hence solve the equation. [10 Marks]

(b) Solve

$$\int_0^x y\phi(y)\mathrm{d}y = f(x)$$

giving reasons for the existence of the solution. [10 Marks]

QUESTION THREE **[20 Marks]**

(a) Convert $f(x) = e^x + \int_0^x (x-y)f(y)\mathrm{d}y$ into a differential equation and hence show that its solution is

$$(\alpha + \beta x)e^x = \gamma e^{-x}$$

where α, β and γ are constants that should be determined.

(b) Solve the following equation:

$$u(x) = e^x + \frac{1}{2}\int_0^1 e^{x-t}u(t)\mathrm{d}t$$

by

(i) the method of successive approximation. [6 Marks]

(ii) letting $C = \int_0^1 e^{-t}u(t)\mathrm{d}t$. [5 Marks]

QUESTION FOUR **[20 Marks]**

(a) (i) Evaluate $\delta_q^p A_s^{qr}$.

(ii) Show that every tensor can be expressed as the sum of two tensors one of which is symmetric and the other skew-symmetric in a pair of covariant or contravariant indices. [9 Marks]

(b) If A^i and B_j are components of a contravariant and covariant tensors respectively of rank one, show that:

$$C_j^i = A^i B_j$$

are the components of a mixed tensor of rank two. [11 Marks]

QUESTION FIVE **[20 Marks]**

(a) Find the Neumann series for the solution of the integral equation:

$$u(x) = (1+x) + \lambda \int_0^x (x-t)u(t)\mathrm{d}t.$$

[10 Marks]

(b) Solve

$$y(x) = x + \lambda \int_0^1 (xz + z^2)y(z)\mathrm{d}z.$$

[10 Marks]