MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 431: NUMERICAL MATHEMATICS II

Date: 13th December, 2016 Time: 3.30 – 6.30 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY)

a) Derive the Laplacian difference equation (4 marks)

b) Explain any TWO types of boundary conditions for solving PDE's and give an example in each case. (4 marks)

c) Classify the following partial differential equations

i) $y^2 u_{xx} - x^2 u_{yy} = 0$

ii) $u_{xx} + 2xu_{xy} + (1-y^2)u_y = 0$

iii) $u_{tt} + 4u_{xx} + 2u_x - u_t = 0$ (6 marks)

d) Write down the Laplace equation for heat flow with sources or sinks of heat within the two dimensional domain. (2 marks)

e) Give the significance of implicit finite difference formulae over explicit finite difference schemes. (4 marks)

f) Solve the following one dimensional Poisson's heat equation

$$T'' = -f(x)$$

where $f(x)$ is a function defining heat source a long a rod and where the ends of the rod are held at fixed temperatures, $T(0,t) = 40$, $T(10,t) = 200$ and $f(x) = 10$ (4 marks)

g) Using Galerkin method of weighted residual, solve the differential equation

$$D(y(x)) = y''(x) + y(x) + 2x(1-x) = 0$$

With the boundary condition:

$y(0) = 0$; $y(1) = 0$ (6 marks)

QUESTION TWO

a) Show that the PDE given below is parabolic everywhere. (3 marks)

$$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$$

b) Solve the Poisson's equation given below using $n = 6$ and $m = 5$ (give the first three solutions only)

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=xe^{y} \qquad 0<x<2, \quad 0<y<1$$

with the boundary conditions

$u(0,y)=0, \qquad u(x,1)=e^{x} \qquad 0\le y\le 1$

$u(x,0)=x, \qquad u(2,y)=2e^{y} \qquad 0\le x\le 2$ (17 marks)

QUESTION THREE

Use explicit finite difference formula to solve for the temperature distribution of a long, thin rod with length 10cm, take coefficient of thermal diffusivity, $k=0.835, \Delta x=2cm, \Delta t=0.1s$. At $t=0$, the temperature of the rod is zero and $T(0,t)=100^0C$, $T(10,t)=50^0C$. Compute the temperatures at the discretized nodes up to $t=0.4$ seconds. (20 marks)

QUESTION FOUR

a) Derive the exact second forward finite-difference formula with error term of order $O(h^2)$, state the error term. (6 marks)

b) Given a triangular element in an x-y plane with nodes(vertices) 1,2,3 and linear equation

$$u(x,y)=a_0+a_{1,1}x+a_{1,2}y$$

Develop equations to approximate the solution for the element. (14 marks)

QUESTION FIVE

Consider a steady-state heat distribution in a thin square metal plate of area one square meters, held at 0 ^{0}C on the two adjacent boundaries while the heat on the other boundaries increases linearly from 0 ^{0}C to 400 ^{0}C where these sides meet. If the sides has zero boundary conditions along the x-axis and y-axis. Taking step sizes to be 0.25, set up the problem and solve for the temperature values numerically for the nine interior grid points. (20 marks)

THE END