TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS UNIVERSITY

EXAMINATION FOR:

BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY & ENVIRONMENTAL PHYSIC BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

APS 4106: WAVES & VIBRATION

END OF SEMESTER EXAMINATION
SERIES: APRIL 2017 TIME
ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consist of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown
This paper consists of FOUR printed pages

Question One (Compulsory)

$$\frac{d^2y}{} = k\,y\,dx^2$$

a) A simple harmonic motion can be described by a differential equation of the form where all the terms have their usual verify that the equation has solution of the form

y□ Acos(kx) □Bsin(kx)

(3 marks)

b) A mass at the end of a spring oscillates with an amplitude of 5cm at a frequency of 1Hz (cycles per second). At t = 0 the mass is at its equilibrium position (x = 0)

(i) Find the possible equation describing the position of the mass as a function of time in the form x □ Acos□ wt □□□ □

What are the numerical values of A, w and ? (6 marks) dx d^2x

__ ____dt dt_2 t □ 83sec

(ii) What are the values of x, and at ? (3 marks)

c) An object of mass 0.2kg is suspended from a spring whose spring constant is 80N/m. The body is subject to a resistive force given by –bv where v is the velocity (m/sec) and b = 4Nm^{-1} sec:

(i) Set up the differential equation of motion for free oscillations of the system and find the period of such oscillation (4 marks)

F□t□ □ Fosin wt

(ii) The object is subjected to a sinusoidal force given by , where F_o = 2N and w 30 sec^{-1}. In the steady state, what is the amplitude of the forced oscillation? (2 marks)

(iii) Instead of a hiring force (in part (ii)), we now oscillate the end of the spring at the top end vertically with a harmonic displacement X = X_o sin (wt). Set up the differential equation of motion for this driven oscillator. (4 marks)

d) A generator of EMF V(t) = V_o cos wt is connected in series with resistance R_1 an inductance L and a capacitance C.

A coin is tossed 3 times. Let X be the random variable denoting the number of heads observed.

(i) Write down the differential equation for the current I in the circuit and for the charge q1 on the capacitor (4 marks)

(ii) Solve for q (w, t) (2 marks)

(iii) Solve for I (w; t) (2 marks)

Question Two

The figure below shows a pulse on a string of length 100m with fixed ends. The pulse is traveling to the right without any change of shape at a speed of 40m/sec D.M

a) Make a clear sketch showing how the transverse velocity of the string varies with distance along the string at the instant when the pulse is in the position shown. (6 marks) b) What is the maximum transverse velocity of the string (approximately) (6 marks)

c) If the total mass of the string is 2kg, what is the tension T in it? (3 marks)

d) Write down an equation for y(x, t) that numerically describes sinusoidal waves of wavelength 5m and amplitude 0.2m travelling in the negative x-direction on a very long string made of the same material and under the same tension as above. (5 marks)

Question Three

a) Two vibrations along the same line are described by the equations:

$x_1 = a\sin w_1 t$

$x_2 = a\sin w_2 t$

(i) Find the beat period of the disturbances. (6 marks)
(ii) Draw a careful sketch of the resultant disturbances (4 marks)

b) Two vibrations at right angles to one another are described by the equations:

$x = a_1 \sin(wt + \varepsilon_1)$

$y = a_2 \sin(wt + \varepsilon_2)$

$\varepsilon_2 - \varepsilon_1 = \pi/2$

Show that if $a_1 = a_2$, and then the projection of the particle will be a circle whose radius is a_1. (10 marks)

Question Four

$X = A\sin(w t + \varepsilon)$

a) Consider a particle attached to a spring executing a motion with A = 0.32m. At t = 0, it is at x = -0.07m and a velocity -2m/s. The total energy is 5.6J. Find:

(i) (3 marks)
(ii) The frequency + (3 marks)
(iii) The spring constant K (3 marks)
(iv) The mass of the particle (3 marks)

b) The displacement from equilibrium, S(t) of the pen of a chart recorder can be modeled as a damped harmonic oscillator satisfying the homogenous differential equation:

$S\ddot{t} + y\dot{s}t + w_0^2 s t = 0$

(i) Find the time evolution of the displacement if the pen is critically damped and subject to the initial condition s (t = 0) and s (t = 0) = Vo (4 marks)

(ii) Show the plot of the critically damped system (4 marks)

Question Five

a) Construct the Lizzojons figures for the following motions. $x = \cos 2wt$, $y = \cos wt$

(i) (2 marks) $x = \cos 3wt$, $y = \cos\left(wt + \frac{\pi}{4}\right)$

(ii) (4 marks)

b) A transverse travelling wave on a cord is represented by D = 0.485m (5.6x + 84t) where D and x are metres and t in seconds. For this wave, determine;

(i) The wavelength (2 marks)

(ii) Frequency (2 marks)

(iii) Velocity (magnitude and direction) (2 marks)

(iv) Amplitude (2 marks)

c) A 440-Hz longitudinal wave in air has a speed of 345m/s

(i) What is the wavelength (1 mark)

(ii) How much time is required for the phase to change by 90^o at a given point in space? (3 marks)

(iii) At a particular instant, what is the phase difference (in degrees) between two points 4.4.cm apart? (2 marks)