MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MAS 207: PROBABILITY AND DISTRIBUTION THEORY II

Date: 29th November, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Start each question a fresh page
- Indicate question numbers clearly at the top of each page.
- Observe further instructions on the answer booklet.

MAS 207: PROBABILITY & DISTRIBUTION THEORY II TIME: 3 HOURS

Instructions:

The paper consists of five questions

Answer **Question 1** and any other two questions

Observe further instructions on the answer booklet

Question 1 (30 Marks)

a) Define the following terms as used in statistics

i) Population

ii) *Random* sample

iii) Sampling distribution of a statistic

iv) Standard error

v) Independent and Identically distributed random variables (10 Marks)

b) Suppose S^2 is the variance of a random sample of size 6 from a normal distribution $N(\mu, 12)$. Determine $p(2.30 < S^2 < 22.2)$ (5 Marks)

c) Let $\bar{X}$ denote the mean of a random sample of size 75 from the distribution which has probability density function $f(x) = 1, 0 < x < 1$ and $0, elsewhere$ Compute an approximate value of $p(0.45 < \bar{X} < 0.55)$ (5 Marks)

d) Let Y denote the sum of items of a random sample of size 12 from a distribution having probability function $f(x) = \frac{1}{6}, x = 1, 2, 3, 4, 5, 6,$ = zero elsewhere. Compute an approximate value of $p(36 \leq Y \leq 48)$ (4 Marks)

e) Let X_i be a random variable distributed, $N(i, i^2), i = 1,2,3$. Assume that the random variables X_1, X_2 and X_3 are independent. Using only the three random variables: X_1, X_2 and X_3. Give an example of a statistic that has:

i) a chi-square distribution with three degrees of freedom.

ii) an F distribution with one and two degrees of freedom.

iii) a t distribution with two degrees of freedom. (6 Marks)

Question 2 (20 Marks)

A population consists of the five values 1, 4, 9, 16 and 25.

i) Calculate the population mean and variance. (3 Marks)

ii) Write down all the samples of size two that may be drawn, with replacement, from this population, and calculate the sample mean of each. (6 Marks)

iii) Let $\bar{X}$ denote the mean of a random sample of size two drawn, with replacement, from this population. Write down the expected value and variance of $\bar{X}$. (3 Marks)

iv) For $\bar{X}$ as in part (iii), find $p(\bar{X} > 16.5)$ Find also an approximation to this quantity, using an appropriate Normal approximation [use of a continuity correction is not expected], and comment briefly on your results. (8 Marks)

Question 3 (20 Marks)

a) State and prove the Chebyshev's theorem (6 Marks)

b) State the difference between Chebyshev's Rule and the Empirical Rule? (2 Marks)

c) It has been determined that the mean return rate for tax-exempt municipal bonds is 9.2% with a standard deviation of 3%. What is the minimum percentage of return rates for tax-exempt municipal bonds with rates between 4.7% and 13.7%? (3 Marks)

d) Let X be a random variable with probability density function

$$f(x) = 630x^4(1-x)^4, 0 < x < 1 \text{ and } = 0 \text{ otherwise}$$

i) obtain the lower bound given by Chebyshev's inequality for $p(0.2 < X < 0.8)$

ii) compute the exact probability, $p(0.2 < X < 0.8)$ (9 Marks)

Question 4 (20 Marks)

a) State and prove the Central Limit Theorem for independent and identically distributed random variables (10 Marks)

b) Distinguish between a sample and a random sample (2 Marks)

c) Define a Simple Random Sample (2 Marks)

d) Briefly describe how to draw a simple random sample using random number tables (6 Marks)

Question 5 (20 Marks)

a) Suppose $X \sim N(0,1)$, using the moment generating function, determine the distribution of X^2

(8 Marks)

b) Given $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with mean μ and variance σ^2, show that

$$\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square distribution with n degrees of freedom. (6 Marks)

c) A fruit-drink company wants to know the variation, as measured by the standard deviation, of the amount of juice in 16-ounce cans. From past experience, it is known that $\sigma^2 = 2$. The company statistician decides to take a sample of 25 cans from the production line and compute the sample variance. Assuming that the sample values may be viewed as a random sample from a normal population, find a value of b such that $p(S^2 > b) = 0.05$

(6 Marks)