MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS, MATHEMATICAL SCIENCE AND MATHEMATICS AND BUSINESS STUDIES WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MAS 303/MMA 314: THEORY OF ESTIMATION

Time: 8.30 - 11.30 am

Date: 29th November, 2016

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.
- Scientific calculators may be used.
- Statistical tables have been appended

QUESTION 1

(a) Clearly distinguish between

(i) **population** and **sample** [2 mks]

(ii) **parameter** and **statistic** [2 mks]

(b) When is an estimator said to be

(i) consistent [1 mk]

(ii) unbiased [1 mk]

(iii) sufficient [1 mk]

(iv) efficient [1 mk]

(c) Explain what's meant by the term **sampling** [1 mk]

(d) Given a random sample of size n from a uniform distribution over the interval $(\alpha \pm \theta\sqrt{3})$, find by the method of moments the estimators for [6 mks]

(i) α

(ii) θ

(e) $f(x;\theta)=\frac{1}{\theta} e^{\frac{-1}{\theta}}$, $x > 0, \theta > 0$

=0 elsewhere

Determine the ML estimator for θ [5 mks]

(f) Show that if T is an **unbiased** estimator for a parameter θ,

(i) a T + b is an **unbiased** estimator for a θ + b [2 mks]

(ii) T^2 **is not an unbiased** estimator for θ^2 [3 mks]

where a and b are known constants

(g) Let $x_1, x_2, \ldots, x_n$ be a random sample from a population with pdf $f(x;\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. Show that $t_i = \prod_{i=1}^{n} x_i$ is a sufficient estimator for θ **[5 mks]**

QUESTION 2

(a) x is a binomial random variable with parameters n and p. Given a random sample of n observations of x, compute the method of moments estimators for

(*i*) p when n is known

(*ii*) n, p when both are unknown **[12 mks]**

(b) (x_1, x_2, x_3) is a random sample of size 3 from a population with mean value μ and variance σ^2. T_1, T_2, T_3 are the estimators used to estimate mean value μ where **[8 mks]**

$T_1 = x_1 + x_2 - x_3$

$T_2 = 2x_1 + 3x_3 - 4x_2$

$T_3 = (\lambda x_1 + x_2 + x_3)/3$

(*i*) Are T_1 and T_2 unbiased estimators

(*ii*) Find the value of λ, such that T_3 is an unbiased estimator for μ

(*iii*) With this value of λ, is T_3 a consistent estimator?

(*iv*) Which is the best estimator and why?

QUESTION 3

(a) State as precisely as possible the properties of maximum likelihood estimators. **[2 mks]**

(b) Obtain the MLE's of α and β for a random sample from the exponential population

$f(x;\alpha,\beta) = y_0 e^{-\beta(x-\alpha)}$, $\alpha \leq x < \infty, \beta > 0$

y_0 being a constant **[12 mks]**

(c) For the density function given by

$f(x;\alpha) = \frac{2}{\alpha^2}(\alpha - x)$, $0 < x < \alpha$

=0 elsewhere

and for a sample of **unit** size,

(*i*) find the MLE for α. **[3 mks]**

(*ii*) show that the estimator is biased **[3 mks]**

QUESTION 4

(a) Prove that a minimum variance unbiased estimator (MVUE) is unique in the sense that if T_1 and T_2 are MVU estimators for $\gamma(\theta)$, then $T_1 = T_2$ almost surely **[8 mks]**

(b) A random sample $(x_1, x_2, \ldots, x_n)$ is taken from a normal population with mean 0 and variance σ^2. Examine if $s^2 = \frac{1}{n}\sum x_i^2$ is a MVUE for σ^2. **[4 mks]**

(c) (*i*) State Crammer-Rao Inequality **[1 mk]**

(*ii*) Let $(x_1, x_2, \ldots, x_n)$ be a random sample from

$f(x;\lambda) = \frac{e^{-\lambda}}{x!}\lambda^x$, $x = 0, 1, \ldots$

=0 elsewhere

By taking $\Psi(\lambda) = e^{-\lambda}$, find Crammer-Rao Lower Bound for t, where t is an unbiased estimator for $\Psi(\lambda)$. **[7 mks]**

QUESTION 5

(a) Explain the problem of interval estimation with special reference to any particular model **[2 mks]**

(b) The tensile strength (x) in Newtons for a certain type of cable was measured for 12 samples. The results were 182, 178, 185, 184, 180, 179, 177, 185, 174, 179, 183, 186. Compute the 90% confidence limits for the mean of x in this type of cable. **[6 mks]**

(c) From the sample values 1.5, 2, .8, 1.3, 2.8, .9, 1.6, 4.2, 3.1, 1.4, 2.2, .7, 1.6, .8, obtain the 95% confidence interval for the population variance assuming that the variable is normally distributed with unknown mean and variance **[6 mks]**.

(d) A random sample of 50 students out of a total of 200 showed a mean of 75 and a SD of 10.

(i) What are the 98% confidence limits for estimates of the mean of the 200 students? **[3 mks]**

(ii) With what degree of confidence could we say that the mean of all the 200 students is 75 ± 1 ? **[3 mks]**