# MASENO UNIVERSITY

## UNIVERSITY EXAMINATIONS 2016/2017

## THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH INFORMATION TECHNOLOGY

## MAIN CAMPUS

## MAS 305: STOCHASTIC PROCESSES I

Date: 8th December, 2016 Time: 12.00 - 3.00 pm

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**INSTRUCTIONS:**

- Answer question ONE and any other TWO questions.
- Observe further instructions on the answer booklet.

## QUESTION ONE (Compulsory) [30 Marks]

(a) Given that $a_n = n \quad n = 0, 1, 2 \ldots,$ find the generating function for the sequence $\{a_n\}$. (3 Marks)

(b) Let X be a random variable such that $Pr(X = k) = p_k$ and $Pr(X \leq k) = q_k = p_k + p_{k-1} + \ldots + p_1 + p_0$.

If

$$P(s) = \sum_{k=0}^{\infty} p_k s^k$$

and

$$\phi(s) = \sum_{k=0}^{\infty} q_k s^k$$

Show that

$$\phi(s)(1 - s) = p(s)$$

(5 Marks)

(c) Let X have a geometric distribution such that $p_k = q^k p, \quad k = 0, 1, 2, \ldots,$ Find the p.g.f, mean and variance of X.

(8 Marks)

(d) Define the following terms

(i) non-recurrent state (1 Mark)

(ii) recurrent state (1 Mark)

(iii) irreducible Markov chain (1 Mark)

(iv) double stochastic matrix (1 Mark)

(e) Classify the states of the following markov chain

$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$$

(10 Marks)

## QUESTION TWO [20 Marks]

(a) Define a probability generating function, and use it to find the mean and variance of the following;

(i) Negative Binomial distribution

(ii) Binomial distribution

(iii) Poisson distribution

(9 Marks)

(b) Let $X$ and $Y$ be two independent random variables with p.g.f's $G_X(s)$ and $G_Y(s)$.

Now let $Z = X + Y$. Find $G_Z(s)$. (6 Marks)

(c) A hen lays N eggs, where N has a Poisson distribution with parameter $\lambda$. Each egg hatches with probability $p$, independent of the other eggs. Find the distribution of Z, the number of chicks.

(5 Marks)

## QUESTION THREE [20 Marks]

(a) Briefly explain the following terms:

(i) covariance stationary process (2 Marks)

(ii) evolutionary process (2 Marks)

(b) Given that $\{x(t), t \in T\}$ is a Poisson process with

$$Pr(x(t) = n) = \frac{e^{-at}(at)^n}{n!}, \qquad a > 0, \quad n = 0, 1, 2, \ldots$$

Prove that $\{x(t)\}$ is evolutionary. (10 Marks)

(c) Let $Y_n$, $n \geq 1$ be uncorrelated random variables with mean 0 and variance 1. Show that the process $\{Y_n, n \geq 1\}$ is covariance stationary. (6 Marks)

## QUESTION FOUR [20 Marks]

## ~~QUESTION FIVE~~ [20 Marks]

(a) Motor vehicles arrive at a petrol pump having one petrol unit, in a Poisson fashion and at an average rate of 10 units per hour. The service is distributed exponentially with a mean of 3 minutes. Evaluate

(i) Average number of units in the system. (2 Marks)

(ii) Average waiting time for a customer. (2 Marks)

(iii) Average length of the queue. (2 Marks)

(iv) Probability that a customer arriving at the pump will have to wait. (2 Marks)

(v) Probability that the number of customers in the system is 2. (2 Marks)

(b) Suppose that whether or not it rains tomorrow depends on previous weather conditions only through whether or not it is raining today. Assume that the probability that it will rain tomorrow given it rains today is $\alpha$ and the probability that it will rain tomorrow given it is not raining today is $\beta$.

(i) If the state space is $S = \{0, 1\}$ where state 0 means it rains and state 1 means it does not rain on a given day. Determine the transition matrix when we model this situation with a Markov chain.

(ii) If we assume there is a 40% chance of rain today, find the probability that it will rain three days from now if $\alpha = \frac{7}{10}$ and $\beta = \frac{3}{10}$.

(10 Marks)

## QUESTION FIVE [20 Marks]

Consider a birth-death process in which the probability of a birth and a death in a small time interval $(t, t + \Delta t)$ is $\lambda \Delta t$ and $\mu \Delta t$, respectively.

(a) Obtain the difference-differential equations for this process. (5 Marks)

(b) Evaluate the mean and variance of $n$. (15 Marks)