MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MAS 401: FURTHER DISTRIBUTION THEORY

Date: 13th December, 2016 Time: 8.30 - 11.30 am

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.

MAS 401: FURTHER DISTRIBUTION THEORY

Answer question one and any other two questions

1) a) The joint probability mass function of a random vector

$$f(x) = \begin{cases} \left(\frac{1}{6}\right)^{x_1} \left(\frac{1}{3}\right)^{x_2} \left(\frac{1}{2}\right)^{x_3} ; & \sum_{i=1}^{3} x_i = 1 \quad , x_i = 0,1 \\ 0 & elsewhere \end{cases}$$

i) Give the support of $\underline{x}$. (2mks)

ii) suppose $D = \{(1,0,0), (0,1,0)\}$ determine $P(\underline{X} \in D)$ (5mks)

b) Explain the meaning of the following:

i). A multinomial random vector

ii). Convergence in quadratic form

c) Let $M(t_1,t_2,\ldots,t_p) = \left(\frac{1}{10}e^{t_1} + \frac{1}{5}e^{t_2} + \frac{3}{10}e^{t_3} + \frac{2}{5}\right)^{30}$ be a joint m.g.f. find the marginal m.g.f of x_2. (6mks)

d) Prove that the covariance matrix can be expressed as $\Sigma = E[\underline{X}\underline{X}^T] - \underline{\mu}\underline{\mu}^T$ (5mks)

e) Suppose $X_1, X_2, \ldots X_n$ are independently and identically distributed random variables with

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 \; elsewhere \end{cases}$$

Find the characteristic function of $Y = x_1 + X_2 + \ldots + X_n$ (5mks)

f) If $X \sim N_p(\mu, \Sigma)$ then show that $x_1, \ldots, x_p$ are independent if and only if Σ is a diagonal matrix i.e. $x_1, \ldots, x_p$ are uncorrelated.

(5mks)

2. a) Suppose $X \sim N_3[\mu, \Sigma]$ having $\mu^T = [6, 3, 4]$ and. Let $\mathbf{X}^T = (\mathbf{X_1}, \mathbf{X_2})$ where $\Sigma = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and Let $\mathbf{X_1} = (X_1, X_2)^T$ and Let $\mathbf{X_3} = X_3$, Determine the distribution of $\mathbf{X_1}$ given $\mathbf{X_2} = 10$

(8mks)

4)a) Suppose x_1, x_2, x_3 are independent random variables having unity variance let

$$Y_1 = 4x_1 + x_2 + 2x_3$$
$$Y_2 = x_1 + 9x_2 - x_3$$
$$Y_3 = 2x_1 - x_2 + 16x_3$$

Find the mean ,covariance and correlation matrices of Y where $Y = [Y_1, Y_2, Y_3]^T$

(10mks)

b) Suppose $X_1, X_2, \ldots X_n$ is a random sample from a population with mean μ and variance σ^2

Let $Q = \sum(x_i - \overline{x})^2$ where $\overline{x} = \frac{1}{n}\sum x_i$.

Show that $E(Q) = (n-1)\sigma^2$ (10mks)

5) a) Let the random variable x be normally distributed with mean μ and variance σ^2, ie $X \sim N(\mu, \sigma^2)$

Derive the characteristic function of X. Hence deduce the moment generating function of x (8mks)

b)Suppose a random variable x is uniformly distributed in the internal [-b,b]. Find its characteristic function. (6mks)

c) Consider the following characteristic function for a random vector X

$\varphi_x(t) = \left[\frac{1}{6}e^{it_1} + \frac{1}{12}e^{it_2} + \frac{1}{3}e^{it_3} + \frac{5}{12}\right]^{12}$ where $t = (t_1, t_2, t_3)$. Determine the characteristic function of

$X_1 = [X_1, X_2]^T$ (6mks)