



MASENO UNIVERSITY
UNIVERSITY EXAMINATIONS 2016/2017

**FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR
THE DEGREE OF MASTER OF SCIENCE IN APPLIED
STATISTICS**

CITY CAMPUS

MAS 804: TEST OF HYPOTHESIS

ate: 2nd December, 2016

Time: 9.00 - 12.00 noon

INSTRUCTIONS:

Answer question ONE and any other TWO questions.

Question 1(24 marks)

(a) Let X have a Bernoulli distribution where $P(x = 1) = \theta = 1 - P(x = 0)$

For a random sample of size $n = 10$

$$\text{Test } H_0: \theta \leq \frac{1}{4}$$

$$H_1: \theta > \frac{1}{4}$$

Use the critical region $\sum x_i \geq 3$

- (i) Find the power function
- (ii) What is the size of this test

(6 marks)

(b) State and prove Neymann Pearson Lemma

(6marks)

(c) Let X have the density

$$f(x, \theta) = \theta x^{\theta-1} \quad 0 \leq \theta \leq 1$$

to test $H_0: \theta \leq 1$

$$H_1: \theta > 1$$

A sample of size 2 was selected and the critical region

(C: $\frac{3}{4}x_1 \leq x_2$) was used.

- Find (i) the power of the test
- (ii) The size of this test

(d) A die was cast 300 times with the following results

Occurrence	1	2	3	4	5	6
Frequency	43	49	56	45	66	41

Are the data consistent at the 0.05 level with the hypothesis that the Die is true?

(6 marks)

Question 2(18 marks)

(a) A thousands individual were classified as to sex and according to whether or not they are colour blind as follows

	Male	Female
Normal	442	514
Colour Blind	38	6

According to the genetic model these numbers should have relative frequencies

	Male	Female
Normal	$p/2$	$p^2/2 + pq$
Colour Blind	$q/2$	$q^2/2$

Where q is the proportion of colour blind individuals in the population. Are the data consistent with the model.

(b) According to the genetic model the proportion of individual having the four blood types should be given by

O: q^2

A: $p^2 + 2pq$

B: $r^2 + 2pr$

AB: $2pr$

Where $p + q + r = 1$

Given

O: 374

A: 436

B: 132

AB:58

Test correctness of the model.

Question 3 (18 marks)

(a) Let x_1, x_2, \dots, x_n be observations from normal population with known variances. How would one test whether their means are all equal.

(9 marks)

(b) Given the sample (1.8, 2.9, 1.4, 1.1) and (5.0, 8.6, 9.2) from a normal population test whether the variances are equal at the 0.05 level.

(9 marks)

Question 4 (18 marks)

(a) Let x_1, x_2, \dots, x_n be a random sample from the Poisson distribution. Find the UMP test of

$$H_0: \theta = \theta_0$$

$$H_1: \theta > \theta_0$$

Sketch the power function for $\theta_0 = 1$ and $n = 25$ (Use central limit theorem)

(b) Let x_1, x_2, \dots, x_n be a random sample from

$$f(x, \theta) = \theta^x (1 - \theta)^{1-x} \quad 0 < x < 1$$

where $\theta = \theta_0$ or $\theta = \theta_1$. We want to test

$$H_0: \theta = \theta_0$$

$$H_1: \theta = \theta_1$$

- (i) show that the test with critical region $\sum x_i \geq k'$ is where k' is a constant. Is a most powerful test.
- (ii) If $\theta_0 = \frac{1}{4}$, $\theta_1 = \frac{3}{4}$, $n = 10$ and $\alpha = 0.0197$ and find k' .

Question 5 (18 marks)

- (a) Let $\Omega = (\theta_0, \theta_1)$. Show that any test arrived at using the generalized likelihood ratio principle is equivalent to simple likelihood ratio test
- (b) Let x_1, x_2, \dots, x_n be a random sample from the uniform distribution over the interval $(\theta, \theta+1)$

To test

$$H_0: \theta = 0$$

$$H_1: \theta > 0$$

The following test was used. Reject H_0 if $y_n \geq 1$ or $Y_1 < k$ where k is a constant

- (i) Determine k so that the test will have size α
- (ii) Find the power function of the test