TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN CIVIL ENGINEERING

SMA 2370: CALCULUS IV

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: JUNE 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) If $\vec{F} = xz^3\hat{i} - 2xyz\hat{j} + 2yz\hat{k}$ find $curl\ \vec{F}$ at (1, -1, 1) **(3 marks)**

b) If $f = f(x, y), x = re^{\theta},\ y = re^{-\theta}$ express $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ in polar coordinates **(7 marks)**

c) Find the directional derivative of $u = 2x^3y - 3y^2z$ at P(1, 2, -1) in a direction toward Q(3, -1, 5) In what direction from P is the directional derivative a maximum and what is its maximum **(7 marks)**

d) Expand $f(x,y) = y^3 / x^3$ in powers of x -1 and y + 1 up to and including second degree terms **(6 marks)**

e) Evaluate $\iiint_v 16z\,dv$ where v is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ **(7 marks)**

Question Two

a) Expand $f(x) = \sin x,\ 0 < x < \pi$ in a Fourier cosine series **(7 marks)**

b) Verify stokes' theorem for $\vec{F} = 2y\,\hat{i} + 3x\,\hat{j} - z^2\,\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 9$ and C is its boundary **(13 marks)**

Question Three

a) Find the equation of the:

(i) Tangent line

(ii) Normal plane to the curve $3x^2y + y^2z = -2,\ 2xz - x^2y = 3$ at the point (1, -1, 1) **(9 marks)**

b) Find the mean value of $\sin mx \sin nx$ over $-\pi < x < \pi$ **(5 marks)**

c) Verify the divergence theorem for the sphere $x^2 + y^2 + z^2 = a^2$ if $\vec{F} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ **(6 marks)**

Question Four

a) Prove that $\int_{(1,2)}^{(3,4)} \left[\left(6xy^2 - y^3\right)dx + \left(6x^2y - 3xy^2\right)dy \right]$ is independent of the path joining (1, 2) and (3, 4). hence or otherwise evaluate the integral **(6 marks)**

b) Test the convergence of $\int_{-\infty}^{\infty} \frac{x^3 + x^2}{x^6 + 1}\,dx$ **(6 marks)**

c) Evaluate $\iint_S (x^2 - xy + y^2)\,ds$ where S is the ellipse given by $x^2 - xy + y^2 = 2$ and using the transformation $x = u\sqrt{2} - v\sqrt{2/3},\ y = u\sqrt{2} + v\sqrt{2/3}$ **(8 marks)**

Question Five

a) The natural frequency of oscillation of an LBC series circuit is given by:

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

If L is increased by 1% C decreased by 1%, show that the percentage increase in f is approximately $\frac{R^2C}{4L - R^2C}$ **(11 marks)**

b) Find and classify all the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$ **(9 marks)**