TECHNICAL UNIVERSITY OF KENYA

DEPARTMENT OF AERNAUTICAL AND AVIATION ENGINEERING

UNIVERSITY EXAMINATION 2016/2017

UNIT CODE: EEAQ 1201

UNIT NAME: MATHEMATICS 1 B: VECTOR ANALYSIS (PART 2).

DATE: APRIL SERIES TIME: 2 HOURS

INSTRUCTIONS:

1) *Answer **Question ONE** and any other **TWO** questions.*
2) ***Mobile phones** and any **written material** are prohibited in the examination room.*
3) *No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.*
4) *All answer booklets should be handed in at the end of the exam whether used or not.*
5) *Programmable calculators are prohibited.*

QUESTION ONE (30 MARKS)

a. Define the term vector quantity and give two examples of vectors (2 mks)

b. Given that **a** =2i + j - 3k, **b** = i - 2j - 5k,c = 3i + j + 2k.Evaluate

i. |a + 2b +3c| (2 mks)

ii. angle between a and b (3 mks)

iii. unit vector parallel to c (2 mks)

c. Given that r = sin t I + cos t j + tk. Find i. $\frac{dr}{dt}$ ii. $\left|\frac{d^2r}{dt^2}\right|$ (3mks)

d. If A=$(2x^2y - x^4)$I + $(e^{xy} - y\,sinx)$j + $(x^2\cos y)$k. Find

i. $\frac{\partial^2 A}{\partial x^2}$ ii) $\frac{\partial^2 A}{\partial y^2}$ iii) $\frac{\partial^2 A}{\partial x\,\partial y}$ (6 mks)

e. If A = $xz^3 i - 2x^2yz\,j + 2yz^4k$ and $\emptyset = 3x^2y - y^3z^2$. Find

(3 mks)

i. $\nabla . A$ at (1,-2,-1) (2 mks)

ii. $\nabla \emptyset$ at (1,-2,-1) (4 mks)

iii. Curl (grad $\emptyset$) or $\nabla \times (\nabla \emptyset)$ at (1, -2, -1)

f. If A = $(3x^2 + 6y)i - 14yzj + 20xz^2 k$. Evaluate $\int A.dr$ from (0, 0, 0) to (1, 1, 1) along the path C: x = t, y = t^2 and z = t^3. (3 mks)

QUESTION TWO (20 MARKS)

a. A particle moves along a curve whose parametric equations are x = 2sin3t, y = 2 cos 3t and z = 3t. Find the velocity and acceleration at time t=0. (4 mks)

b. If A = $x^2yzi - 2xz^3 j + 3xz^2k$ and $B = 2z\,i + yj - 3x^2k$. Find

i. $\frac{\partial(A.B)}{\partial x}$ at (1,0,-2) (3 mks)

ii. $\frac{\partial^2(A \times B)}{\partial x \partial y}$ at (1,0,-2) (4 mks)

c. Consider a vector V given by V = (x + 2y + az)I + (bx – 3y -z)j + (4x + cy +2z)k. Find constants a, b, c so that V is irrotational / conservative. Hence find the scalar potential $\emptyset$ such that V = $\nabla\emptyset$ (9 mks)

QUESTION THREE (20MARKS)

a. Given the space curve x=3 cos t,y = 3 sin t, z =4t.Find

i. Curvature ii. Radius of curvature iii. Torsion (12 mks)

b.if $\emptyset = 2xyz^2$, F=xy I – zj + x^2k and C is the curve x = $t^2, y = 2t, z = t^3$ from t = 0 and t = 1.Evaluate i. $\int \emptyset$ dr (3 mks)

ii. $\int F \times dr$ (5 mks)

QUESTION FOUR (20MARKS)

a. The acceleration of a particle at any time t is given by a = $e^{-t}i - 6(t+1)j + 3\sin t\, k$. If the velocity v and displacement r are both zero at t =0.Find v and r at any time (6 mks)

b. Evaluate $\int F.dr$ where F=(x – 3y)i+(y – 2x)j and C Is the closed curve in the xy plane, x=2 cos t, y = 3 sint from t=0 and t=2π if C is traversed in the anticlockwise direction. (8 mks)

c. Use Green's theorem to evaluate $\int (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by y = x and y = x^2 (6 mks)

QUESTION FIVE (20MARKS)

a. Evaluate $\iint_S A.n\, ds$ where A = 18z i – 12 j +3yx k and S is part of the plane 2x + 3y + 6z =12 which is located in the first octant. (10 mks)

b. Evaluate $\iint_S F.n\,ds$ where F = 4xz I $- y^2\ j$ + yz k and S is the cube bounded by x =0, x = 1, y = 0, y = 1, z = 0, z = 1 using divergence theorem (5 mks)

c. Use divergence theorem to evaluate $\iint_S A.n\,ds$ given that A = 4x i - $2y^2\ j$ + $z^2 k$ taken over the region bounded by $x^2 + y^2 = 4$, z = 0 and z = 4. (5 mks)