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**University Examinations 2015/2016**

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

**SMA 2404: FIELD THEORY**

**DATE: November, 2015 TIME: HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE – (30 MARKS)**

1. (i) Define an integral domain. (1 Mark)

(ii) Show that every field is an integral domain. (4 Marks)

1. Express in set builder notation. Show why is not regarded as a field even though is a field. (4 Marks)
2. The polynomial can be regarded as sequence which mapping defined by

f(0) = ,f(1), = = ………. f(n) = , f(i) = 0 for i

+ as a sequence and express this sequence as a mapping

1. Marks)
2. The element said to be algebraic over F if satisfies a polynomial equation over f (where f is a field) Show that , i = is algebraic over . (5 Marks)
3. (i) Write out Qin a set builder notation. (1 Mark)

(ii) Show that if K = then K = Q) (3 Marks)

1. (i) Define characteristics of a field. (2 Marks)

(ii) Show that if F is a field with characteristics n then n is prime number(3 Marks)

1. Determine whether or not f(x) = is irreducible over (3 Marks)

**QUESTION TWO – (20 MARKS)**

1. Show that a field F is algebraically closed if every non-constant polynomial f(x) has factorization into linear factors. (8 Marks)
2. If E is an algebraic extension of a field F. Show that there exists a finite number of elements …… in E such that E = F(…..) If and only if E is a finite dimensional space over F. (6 Marks)
3. Show that a finite extension field E of a field F is an algebraic extension of F. (6 Marks)

**QUESTION THREE (20 MARKS)**

1. Find the polynomial in (x) with + as root. (6 Marks)
2. Let be irreducible polynomial over the field F. Show that F has a finite extension field K in which has a root. (7 Marks)
3. Let IR. If f(a+bi) = 0 , for (a + bi) where a, b. show that

(7 Marks)

**QUESTION FOUR (20 MARKS)**

Let f be subfield of a field , and x be indeterminate.

1. Show that the map (……++……. for +…… is a homorphism of into (show that is well defined)

(10 Marks)

1. Prove that maps F isomorphically by identity map ,for a (3 Marks)
2. Draw the addition and multiplication table for;

(7 Marks)

**QUESTION FIVE (20 MARKS)**

1. Let f(x) and an extension of F. What does it mean to say that;
2. “f(x) splits in ”
3. “E is a splitting field for f(x) over F”

Give example (6 Marks)

1. Find the splitting field for each polynomial below:
2. over (5 Marks)
3. (3 Marks)
4. Show that an element a in a field F is a zero of f(x) F if x – a is a factor of f(x) in F (7 Marks)