JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE WITH IT

4TH YEAR 2ND SEMESTER 2017/18 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SPH 402

COURSE TITLE: STATISTICAL MECHANICS

EXAM VENUE: **STREAM: (BED SCI)**

DATE: 21/05/2018 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2:00HRS

Instructions:

1. **Answer question 1 (Compulsory) in Section A and ANY other 2 questions in Section B.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Useful constants

Planck's constant = 6.63×10^{-34} J s

Velocity of light in free space = 3.0×10^{8} m/s

Boltzmann's constant = 1.38×10^{-23} J/

Question 1

(a) Explain the meaning of the following terms:

(i) Accessible microstate

(ii) Phase space.

(iii) Canonical ensemble.

(3 Marks)

(b)Find the number of quantum states which are accessible to a system if its entropy is 42 J/K.

(2 Marks)

(c) Two distinguishable particles are in an energy level with degeneracy 4. Determine the number of microstates of these particles.

(3 Marks)

(d) Derive the Stirling's approximation for N!

(4 Marks)

(e) Use expressions for internal energy, U, and Helmholtz free energy F to obtain chemical potential as a function of F.

(4 Marks)

(f) Gibb's free energy is applicable in diffusive processes that reach equilibrium under isothermal and isobaric constraints. Considering two systems that are interacting diffusively, obtain an expression for the change in Gibbs free energy for the combined system, starting from the definition of Gibbs free energy.

(5 Marks)

(g) Explain an example of a situation where one can encounter negative temperature. Why does negative temperature not occur in ordinary systems?

(3 Marks)

(h) Estimate the density of states accessible to an air molecule in a classroom of dimensions 5 m by 9 m by 4 m given that the molecule's maximum energy is about 3×10−21 J and its mass is 5.6×10−26 kg.

(3 Marks)

(i) Show that the Bose-Einstein distribution reduces to the Planck's distribution, in the case of photons.

(3 Marks)

Question 2

(a) Consider five small interacting systems for which Ω1 = 1, Ω2 = 2, Ω3 = 3, Ω4 = 4, and Ω5 = 5. (i) What is the number of states Ω0 accessible to the combined system? (ii) What are the entropies S1 , S2 , S3 , S4 , and S5 in terms of Boltzmann's constant k? (ii) Compute the entropy of the combined system S0 in units of Boltzmann's constant.

(2, 5, 3 Marks)

(b) In the chemical reaction A + B → C, the molar heat capacities of the reactants (at constant pressure) in units of J/(mole K) are CA = 8√T, CB = 9√T, CC = 15√T, where T is in kelvins. If this reaction is carried out at 400 K, how much heat is absorbed or released if one mole of substance C is produced? (10 Marks)

Question 3

(a) A system of 7 particles that obey Maxwell-Boltzmann statistics has two energy levels, one with a degeneracy of 3 and the other degeneracy of 4. (i) List down all the macrostates in the system in terms of the number of particles in each level. (ii) For any three specified macrostates, determine the number of microstates in the system.

(3, 7 Marks)

(b) Given the accessible states in a Maxwell-Boltzmann distribution

Ω = N! Πj

gnj j nj!

$\Omega = N! \prod_j \frac{g_j^{n_j}}{n_j!}$

show that the Maxwell-Boltzmann distribution function is given by

nj =

N Z

gje−β j

$n_j = \frac{N}{Z} g_j e^{-\beta \epsilon_j}$

where the symbols have their usual meanings.

(10 Marks)

Question 4

(a) A box contains an ideal classical gas at pressure p and temperature T. The walls of the box have N0 absorbing sites, each of which can absorb one molecule of the gas. Let −ϵ be the energy of an absorbed molecule. (i) Find the fugacity of the gas in terms of temperature and pressure. (5 Marks)

3(ii) Find the mean number of absorbed molecules ~~hNi~~. $\langle N \rangle$ (5 Marks)

(b) Show that the total energy of blackbody radiation is proportional to the fourth power of the absolute temperature T.

(10 Marks)

Question 5

(a) An ideal Bose gas is composed of N particles in a volume V . The number of particles in the lowest one-particle state and the number of particles in the higher states are N0 and N′ respectively. Show that when the temperature falls below a value Tc, N0 suddenly becomes comparable with N. (15 Marks)

(b) A system consisting of N non-interacting atoms in a substance at absolute temperature T is placed in an external magnetic field pointing along the z−direction. Derive an expression for the z−component of its magnetic moment μz.

(5 Marks)