UNIVERSITY OF KABIANGA
SCHOOL OF BUSINESS AND ECONOMICS

UNIVERSITY EXAMINATIONS 2017/2018

THIRD YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF ARTS IN ECONOMICS AND BACHELOR OF EDUCATION ARTS

ECO 312: MATHEMATICS FOR ECONOMIST

DATE: FEB. 2018

DURATION: 3 HRS

Instructions: Answer Question **ONE** and other **THREE**. Answer Each on a New Page.

Question One [25 Marks]

[a.] In a laboratory experiment, the rate of change of population of bacteria ***P(t)*** with time ***(t)*** is given by the following. $\frac{dP(t)}{dt} = 0.045[900 - P(t)]$

Given that P(0) = 0, find the following;

[i.] The time path **P(t)** **[1 Mark]**

[ii.] The rate of growth of the population of bacteria At $t = 0$ and at $t = 10$ **[1 Mark]**

[iii.] The steady state (inter-temporal equilibrium) value of **P(t)** **[1 Mark]**

[iv.] Provide the graphical sketch of time path of the population of bacteria **[3 Marks]**

[b.] Given the following constrained optimization problem

Maximize $Z = 2x + 6y - 3y^2$

Subject to $2x - 2y^2 = 8$

[i.] Find the critical values of x, y and λ **[1.5 Marks]**

[ii.] Do the critical values present a maximum or a minimum?

 scarce resources = optimal use

Apply Bordered Hessian method. **[2 Marks]**

[iii.] What are the extreme values of **Z** and **L** (Langrangian function)? **[2 Marks]**

[c.] Explain the following terms;

[i.] Dynamic stability

[ii.] Steady state **[2 Marks]**

[d.] Discuss the dynamic stability of the following time path.

[i.] $y(t) = 5e^{-3t} + 15$

[ii.] $y(t) = 3e^{-at} + 5$, where a = -2

[iii.] $y(t) = 4e^{-0.3t} + 8$ **[4.5 Marks]**

[e.] A monopolistic firm has the following demand and total cost functions

$$P = 100 - 2Q$$
$$TC = \frac{1}{2}Q^3 - 9Q^2 + 120Q + 175$$

Required;

[i.] The **TR** and π functions **[2 Marks]**

[ii.] The profit maximizing level of **Q**. Confirm that second order condition is satisfied. **[1 Mark]**

[iii.] The **MC** and **MR** at the profit maximizing level of **Q**. Compare their magnitudes. **[2 Marks]**

[iv.] The value of **Q** that maximizes **TR**. Confirm that second order condition is satisfied. **[2 Marks]**

Question Two [15 Marks]

[a.] Consider the following demand function

$$Q_a = 300 - 3P_a + 2P_r + 0.2Y \text{ where } P_a = 20;\ P_r = 30;\ \text{and } Y = 800$$

Determine the following and interpret the results:

[i.] Own price elasticity of demand **[1.5 Mark]**

[ii.] Cross-price elasticity of demand. **[1 Mark]**

[iii.] Income elasticity of demand. **[1.5 Mark]**

[iv.] Are the two goods (**a** and**r**) substitutes or compleme[ments?]
Explain. [2 Ma[rks]]

[v.] Is good **a** normal or inferior good? Explain. [1Mar[k]]

[b.] State and demonstrate Euler's Theorem [2 Marks]

[c.] A consumer has the following utility function

$$U = Ax^{\alpha}y^{\beta}$$

Show that

[i.] $x\frac{\partial U}{\partial x} + y\frac{\partial u}{\partial y} = (\alpha+\beta)U$ [2 Marks]

[ii.] $\frac{dy}{dx} = \frac{\alpha y}{\beta x}$ [2 Marks]

[iii.] Given that$\partial U = 0$, Find that MRCS for the utility function. [2 Mark]

Question Three [15 Marks]

[a.] Given the following bivariate equations, compute the partial derivatives, second order partial derivatives and cross partial derivatives. Comment on the cross partial derivatives.

[i.] $Z = f[x, y] = [3x^3 + 4y^6 - y^4]^8$

[ii.] $Z = \left[\frac{4x^2 + 7}{x^2 + y^2}\right]$ [5 Marks]

[b.] Explain the term returns to scale? [4 Marks]

[c.] Find the degree of homogeneity of the following production function and state whether the functions display constant, increasing or decreasing returns to scale.

[i.] $Q = AK^{\alpha}L^{n-\alpha}$

[ii.] $Q = AK^{\frac{4}{7}}L^{\frac{4}{7}}$ [6 Marks]

Question Four [15 Marks]

 scarce resources = optimal use

Consider the following bivariate Utility function.

$$U = 25x^{\frac{2}{5}}y^{\frac{3}{5}}$$

Evaluate the following;

[i.] The MU_x and MU_y **[1 Mark]**

[ii.] **MRCS** between the two goods **[1 Mark]**

[iii.] By setting **U = 100**, derive the corresponding indifference curve (express y in terms of x). **[2 Marks]**

[iv.] The **MRCS** for x = 2 **[1 Mark]**

[v.] Does the indifference curve obey the law of diminishing **MRCS**? **[1 Mark]**

[b.] Given the following differential equation and the accompanying initial conditions.

$$\dot{y} + 3y = 12 \quad \text{(and at t=0, then y = 24)}$$

Required;

[i.] the homogeneous solution

[ii.] the particular solution

[iii.] the general solution

[iv.] the unique or definite solution **[4 Marks]**

[c.] For each of the following functions,

[g1.] $y = f[x] = -(x + 3)^4$ and [g2.] $y = f[x] = x^3 + 7$

Find;

[i.] The critical values of **x** **[2 Marks]**

[ii.] By applying the ***rth*** derivative test, determine whether the critical values yields a relative maximum, relative minimum or an inflexion point. **[3 Marks]**

Question Five [15 Marks]

[a.] Using relevant example, distinguish between a structural and reduced form equations **[3 Marks]**

[b.] A firm has the following demand and supply functions in a market where equilibrium prevails.

$$P = Q_s + 50$$
$$P = -4Q_d + 100$$

Further, the government imposes a per unit tax **'t'** on the product supplied. With the aid of an appropriate diagram, find the level of **'t'** that will maximize tax revenue **[5 Mar**

[c.] A multiproduct firm has the following cost function and productio quarter that must be observed.

$$C = Q_1^2 + Q_1Q_2 + 2Q_2^2 + 20$$

$$Q_1 + Q_2 = 8$$

Required;

[i.] Set up a constrained cost minimization problem from the information given **[1 Mark]**

[ii.] Construct the corresponding Langrangian function **[L]**.**[1 Mk]**

[iii.] Find $\overline{Q_1}$, $\overline{Q_2}$ and λ by applying the first order condition. **[3Mks]**

[iv.] Confirm that the critical values present minimum costs. **[1Mark]**

[v.] What are the stationary values o **C** and **L**? **[1 Mark]**

