QUESTION ONE (30 Marks)

a) Define the following terms
i) Power set
ii) Predicate
iii) Composite function **(3 marks)**

b) Let $A = \{ x : x = 2k + 1,\ 0 \leq k \leq 6,\ k \in \mathbf{Z} \}$, and $B = \{ 3, 8, 13, 18, 23 \}$.
Determine;
i) $n(A)$, ii) $A \cap B$, iii) $A - B$, iv) $A \Delta B$, iv) B in setbuilder form.
(6 marks)

c) i) State the Trichotomy law of integers.
ii) Solve the inequality $n^2 - 3n - 4 > 0$ where $n \in \mathbf{Z}$.
(4 marks)

d) i) Define a rational number
ii) If $x = 3 + \sqrt{5}$, express as a function with rational denominator $x - x^{-1}$.
(4 marks)

e) Given the functions;
$f = \{ (a, 3), (b, 2), (c,1), (d, 5)\}$, $g(x) = 4x^2 - 1$, and $h(x) = e^x$ where
$g: \mathbf{R} \rightarrow \mathbf{R}$ and $f: \mathbf{R} \rightarrow {}^{+}\mathbf{R}$
i) state giving reasons which of these functions are not surjective,
ii) write down where possible the inverse of these function,
iii) for the function(s) in (ii) without an inverse find a suitable domain so that inverse exists and write it down. **(6 marks)**

f) State, with reasons which of the following statements are not prepositions
(i) Has she solved the problem?
(ii) $\{ x : 4x - 1 = 0 \}$
(iii) $\forall_{x \in N} \ (x^2 + 5x + 6) = 0$
(2 marks)

g) For the statement**:** "If a quadrilateral is a square then it is a rectangle";
State the;
i) converse, ii) inverse iii) contrapositive , and iv) negation.
(5 marks)

QUESTION TWO (20 Marks)

a) Let $A = \{1, 2, 3\}$ and $B = \{ a, b \}$
Find : i) $P(B)$ ii) $A \times B$
(2 marks)

b) Use algebra of sets to prove that $(A' \cap B)' \cap (A \cup B')' = B'$
(4 marks)

c) Write down the setbuilder definition for $A \Delta B$.
Hence show that
$(A \Delta B) \subseteq [(A - B) \cup (B - A)]$. **(4 marks)**

d) Of 150 home with television sets, on one particular evening 50 watched KBC only, 60 watched KTN only and 5 watched NTV only. Ten watched both NTV and KTN.
Twenty watched both KBC and KTN, and 15 watched both KBC and NTV.
If equal numbers watched all three as watched none, use a Venn diagram to find how many watched none?

(10 marks)

QUESTION THREE (20 Marks)

a) Prove by mathematical induction that for all $n \in N$

$$3(6) + 6(9) + 9(12) + \ldots + 3n(3n+3) = 3n (n+1) (n+2).$$

(6 marks)

b) Let x, y, and z be integers and $x = y \pm z$. Show that if $a \mid x$ and $a \mid y$, then $a \mid z$.
(5 marks)

c) i) Find a counterexample to disprove the statement that:
$2n^2 + 11$ is prime.
ii) Given that:
"If $3n + 2$ is odd, then n is odd" , construct a contrapositive proof of the statement. **(9 marks)**

QUESTION FOUR (20 Marks)

a) Let $f(x) = 1/x$, $x \neq 0$ and $g(x) = x^2 - 1$ where $x \in \mathbf{R}$
Find;
i) $(fg)(x)$ by and $(gf)(x)$,
ii) Domain of fg,
iii) $g(f(-5))$.

(6marks)

b) If $\sqrt{x^2 + 1} = f(g(h(x)))$, determine h, g and f.

(2 marks)

c) Given $f(x) = x^2 - 4x + 7$, $x \in \mathbf{R}$,determine;
i) if f(x) is one – to – one
ii) if f(x) is onto
iii) a restricted domain so that $f^{-1}(x)$ exists and find it
iv) the domain and the range of the inverse

(12 marks)

QUESTION FIVE (20 Marks)

a) Determine using a truth table which of the following propositions are logically true, false or indeterminate

i) $p \wedge \sim p$ ii) $p \vee \sim p$ iii) $p \vee \sim q$

iv) $(p \rightarrow q) \vee (q \rightarrow p)$ **(7 marks)**

b) Use a truth table to show that;

$[(p \rightarrow q) \text{ AND } (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology

(5 marks)

d) Given the statement:

"There exists a real number x such that if $x > 1$, then $3x^2 - 7x + 2 = 0$",

i) Determine the truth set and the truth value of this statement.

Rewrite this statement:

ii) using existential logic symbols

iii) starting with the phrase;

" For all… "

(8 marks)