

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SMA 260:PROBABILITY AND STATISTICS I

PAST PAPER.

QUESTION ONE

- a) Outline four characteristics of the normal probability distribution. (2mks)
b) A discrete random variable x has a probability function given by:

x	0	1	2	3	4	5
$f(x)$	0.10	0.15	0.30	0.25	0.15	0.05

- i) Show that the function $f(x)$ is a probability mass function(p.m.f) (2mks)
ii) Determine the mean and variance of the random variable x in the probability distribution. (6mks)
iii) Determine the probability that x is at least 3. (2mks)
- c) A discrete random variable x has a Poisson probability distribution with probability mass function(p.m.f) given by:

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & \text{for } x = 0, 1, 2, 3, \dots, n. \\ 0 & ; \text{ elsewhere} \end{cases}$$

Prove that the mean of x and the variance of x are both equal to λ (6mks)

- d) A continuous random variable x is given by the function.

$$f(x) = \begin{cases} \frac{1}{2} x & ; \text{ for } x = 0, 1, 2, 3, \dots, n \\ 0 & ; \text{ elsewhere} \end{cases}$$

i) Show that the function $f(x)$ is a probability density function(p.d.f)

ii) Hence determine the probability: $P(\frac{1}{2} \leq x \leq 1)$. (4mks)

e) A continuous random variable x given by the probability density function

$$f(x) = \begin{cases} \frac{1}{2}(x+1); & \text{for } -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Determine the variance of $(5x + 10)$ (4mks)

f) A continuous random variable x is normally distributed with parameters mean $\mu = 10$ and variance $\sigma^2 = 36$.

Determine the the probability: $P(4 \leq x \leq 6)$. (4mks)

QUESTION TWO

A continuous random variable x is given by the function

$$f(x) = \begin{cases} \frac{2}{9}(9x - x^2 - 18) & ; \text{for } 3 \leq x \leq 6 \\ 0 & \text{elsewhere} \end{cases}$$

a) Show that the function $f(x)$ is a probability density function(p.d.f) (3mks)

b) Determine the following measures about the random variable x in the probability distribution;

i) The mode of x ;

ii) The mean of x ;

iii) The variance of x ; (13mks)

Determine the following probability from the distribution:

$P(2 \leq x \leq 5)$ (4mks)

QUESTION THREE

a) The national transport and safety authority claims that only 60% of the drivers on the Kenyan roads are trained in driving schools. A random sample of 12 drivers on the road was taken from Kenya. Determine the probability that among these drivers the following were trained in a driving school:

i) exactly 5

ii) At least 4 drivers

iii) Between 4 and 6 drivers inclusive (10mks)

b) The daily water usage of residents of Thika town has been found to be normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons:

i) Determine the probability that a resident selected at a random will use less than 15 gallons of water per day;(3mks)

ii) Determine the probability that the resident selected at random will use more than 30 gallons of water per day;(3mks)

iii) Determine the percentage proportion of Thika residents who use between 25 and 30 gallons per day. (4mks)

QUESTION FOUR

- a) Outline three characteristics of the binomial probability distribution. (3mks)
- b) The number of people who become ill each year from eating a kind of poisonous plant in a certain region has been found to have a Poisson probability distribution with rate parameter $\lambda = 1.6$. Determine the probability of obtaining at random the following number of people who have fallen ill from the plant in a given year:
- i) Exactly 2 people;
 - ii) At least 7 people;
- (7mks)
- c) The operational lifespan of a given brand of a photocopying machines has been found to be normally distributed with a mean of 4.8 years and a standard deviation of 1.6 years.
- i) Determine the proportion of these photocopying machines that will have a lifespan of between 3.8 years and 6.6 years;
 - ii) if this photocopying machines have a warranty period of 2 years, determine the proportion that the original sales which will require replacement through this warranty.
 - iii) If the manufacturer of these photocopying machines wants only 5% of the machines to be replaced through this warranty, Determine the warranty period that should be set to achieve this.
- (4mks)
(3mks)
(3mks)

QUESTION FIVE

- a) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} xe^{-x}; & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

- i) Determine the moment generating function of x .
 - ii) Hence determine the mean and variance of x .
- (10mks)

- b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x; & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- i) Determine the probability density function of a continuous random variable $y = 8x^3$ using the change of variable technique.
 - ii) Hence determine the following:
 - I. The mean of y
 - II. The probability $p(y > 4)$
- (5mks)
(5mks)

