

**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS**  
**2011/2012 ACADEMIC YEAR**  
**FOR THE DEGREE OF BACHELOR OF SCIENCE IN**  
**TELECOMMUNICATION**

**MATH 210: ENGINEERING MATHEMATICS II**

**DAY: WEDNESDAY**

**DATE: 01/08/2012**

**TIME: 9.00 – 11.00 A.M.**

**STREAM: Y2S1**

**INSTRUCTIONS:**

- **Answer Question ONE and any other TWO Questions**

**QUESTION ONE (30 MARKS)**

- a) Let the matrix  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and the matrix  $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ . Find the matrix  $A^{-1}B$  and hence or otherwise solve the system  $Ax = B$ . (8 marks)
- b) State De Moivre's theorem for positive integral index and use it to prove that  $\sin 3\phi = 3\cos^2 \phi \sin \phi - \sin^3 \phi$ . (5 marks)
- c) Find the volume of the Tetrahedron with sides  $a, b, c$  given by  $a^2 + b^2 = c^2$  and  $a, b, c > 0$ . (6 marks)
- d) Let  $\dim_{\mathbb{R}} V = 2$  and define  $T$  on  $V$  by  $v_1 T = av_1 + bv_2$  and  $v_2 T = xv_1 + yv_2$ .

where  $a, b, x, y \in \mathbb{R}$ . In terms of  $a, b, x, y$ , find necessary and sufficient conditions that  $T$  have two distinct eigenvalues in  $\mathbb{R}$ . (5 marks)

e) Find all the 5<sup>th</sup> roots of unity (6 marks)

**QUESTION TWO (20 MARKS)**

a) Let the mapping  $f$  and  $g$  defined by  $f(x, y) = (x^2 + y^2, x^2 - y^2)$  and  $g(x, y) = (x^2 + y^2, x^2 + y^2)$ . Find  $f \circ g$ .

- i.  $f \circ g(2, 5)$
- ii.  $f \circ g(5, 2)$
- iii.  $f(f(2, 5))$
- iv.  $g(g(2, 5))$  (9 marks)

b) Find the area of a triangle that has the vertices  $(1, 2, 3)$ ,  $(2, 3, 4)$  and  $(3, 4, 5)$ . (5 marks)

c) If  $\tilde{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$  find  $\nabla \times \tilde{A}$  at  $(1, -1, 1)$ . (6 marks)

**QUESTION THREE (20 MARKS)**

a) Evaluate  $\int_C \frac{(1-2z)dz}{z(z-1)(z-2)^2}$  where  $C$  is a positively oriented circle  $|z| = 1.5$ . (5 marks)

b) Let  $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined by  $F(x, y) = (2x + 3y, 4x - 5y)$ . Find the T of F relative to the basis  $S = (u_1, u_2) = \{(1, -2), (2, -5)\}$  (6 marks)

c) Consider the space curve  $x = 3\cos t$ ,  $y = 3\sin t$ ,  $z = 4t$ .

- Find
- i). The unit tangent  $\hat{T}$
  - ii). The principal normal  $\hat{N}$
  - iii). Curvature  $k$  and radius of curvature  $\rho$
  - iv). The Binormal  $\hat{B}$
  - v). Torsion  $\tau$  and radius of torsion  $\sigma$  (9 marks)

**QUESTION FOUR (20 MARKS)**

a) Use Cramers rule to solve the system

(12 marks)

b) If  $\tilde{A} = \cos xy\hat{i} + (3xy - 2x^2)\hat{j} - (3x + 2y)\hat{k}$  show that ——— (4 marks)

c) If  $F(z) = \frac{3z+1}{(z-4)(z-1)}$ , find the poles and residues at the poles for  $F(z)$ . (4 marks)

**QUESTION FIVE (20 MARKS)**

a) Show that if ——— is analytic inside and on a simple closed curve C and  $a$  is any point inside C then ——— (7 marks)

b) A particle moves along the curve  $\tilde{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ , where t is the time. Find the magnitude of the tangential and normal components of its acceleration when ———.

c) Given that matrix  $A = \begin{pmatrix} -7 & 3 \\ 5 & 1 \end{pmatrix}$  and matrix  $B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ , find ——— where;