KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2013/2014 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MATH 113: CALCULUS 1

DAY: SATURDAY **DATE: 19/04/2014**

TIME: 10.00 - 1.00 P.M. **STREAM: Y1S1**

INSTRUCTIONS:

- Answer question **ONE** and any other **TWO**

QUESTION ONE (30MARKS)

a. Find the limits of the following

i. $\lim_{x\to 2} 3 - x^2 + 5x$ (2Mks)

ii. $\lim_{x\to 1} \frac{x^2+x-2}{x^2-x}$ (2Mks)

iii. $\lim_{x\to \infty} \frac{5x^2+8x-3}{3x^2+2}$ (2Mks)

b. Use first principles to differentiate (2Mks)

i. y=x^2 (2Mks)

ii. y=$2x^2$-x+5 (3Mks)

c. By using chain rule find $\frac{dy}{dx}$ given that;

i. y=$(x^2 + 2x)^{-2}$ (3Mks)

ii. Use the second derivative` to determine whether the function y=$x^3 - 6x^2 + 9x + 2$ has a minimum or a maximum.

(3Mks)

d. Find the equations of the tangent and normal to the curve y=$x^3 + 8x + 5$ at x=2

(4Mks)

e) (i)The curve y=$x^2 + ax + b$ has a turning point at (1,3). Find the value of a and b. (2Mks)

(ii) Find the derivative f(x)=$\frac{\ln x}{\sin x}$ (2Mks)

f) Find the equation of the tangent to the circle x^2+4y^2=80 at the point (1,1) (3Mks)

QUESTION TWO (20MARKS)

a. Find the gradient of the curve:'

i. x^2+2xy-$2y^2$+x=2 at the point(-4,1) (3Mks)

ii. x=$\frac{t}{1+t}$, y=$\frac{t^3}{1+t}$ at the point ($\frac{1}{2}$, $\frac{1}{2}$) (3Mks)

b. Find the $\lim_{x\to\infty}\sqrt{\frac{8x^2-3}{2x^2+x}}$ (3Mks)

c. Find the value of y' at point (-1,1) of the function x^2y+3y-4=0 (4Mks)

d. Show that if f(x)=$\frac{\sin x}{\cos x}$ then f'(x) =sec^2x (3Mks)

e. Investigate the stationary values of the function f(x)=x^4-4x^3 (4Mks)

QUESTION THREE (20 MARKS)

a. A container in the shape of right circular cone of height 10cm and base radius 1cm is catching the drips from a tap leaking at 0.1 cm^3/s. find the rate at which the surface area is increasing when water is half way up the cone. (7Mks)

b. (i) Find the derivative of y=$\frac{t^2-1}{t^3+1}$ (3Mks)

(ii) Differentiate y=sin(2x+3) (2Mks)

(iii) Find $\frac{dy}{dx}$ if y=$x^2e^x - 2xe^x + 2e^x$ (3Mks)

c. A farmer has 10,000m of fencing wire with much to fence three of his rectangular farm. The fourth side being an existing fence of his neighbour. Find in metres the dimension of the field of the largest possible area that can be enclosed. (5Mks)

QUESTION FOUR (20MARKS)

a. Find the equation of the tangent line to the circle $x^2 + y^2$=25 at a point (3,4)

(4Mks)

b. Write the equation of the tangent and normal of the equation $x^5 + y^5 - 2xy = 0$ at a point (1,1) (6Mks)

c. Differentiate:

i. y=$x^2 lnx$ (2Mks)

ii. y=$\frac{e^x}{\sin x}$ (3Mks)

d) (i) Suppose a spherical balloon is inflated at the rate of 2cm^3/s. what is the rate of growth of the radius. (3Mks)

(ii) Differentiate y=$cos^2 x$ (2Mks)

QUESTION FIVE (20MARKS)

a) (i) Evaluate $\lim_{x\to\infty} \frac{4}{x^{\frac{2}{5}}}$ (2Mks)

(ii).Differentiate using first principles y=$x^{1/2}$ (3Mks)

b) (i) Find the coordinates of the points on the curve y=x^3-2x+7 at which the gradient is 1.

(3Mks)

(ii) Find the turning values of y on the graph y=5+24x-$9x^2$-$2x^3$ (4Mks)

c) State the necessary condition for a function f(x) to be continuous (3Mks)

d) A particle moves from a point Q so that after t second it is S metres from Q. where

S =8t-t^2.

Find;

i. The velocity of the particle at t=0 and t=4 seconds (3Mks)

ii. The acceleration of the particle at any time (2Mks)