KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS
MAIN CAMPUS

THIRD SEMESTER, 2015/2016 ACADEMIC YEAR

EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

MATH: 113 CALCULUS

STREAM: Y1S1 **TIME: 3.00 – 5.00 PM**

EXAMINATION SESSION: AUGUST **DATE: 11/8/2016**

INSTRUCTIONS:

Answer question **one** and any other **two**

QUESTION ONE (30MARKS)

a) Evaluate the following limits:

i. $\lim\limits_{x\to 2}\dfrac{x^2-7x+10}{x^2-4}$ (2 marks)

ii. $\lim\limits_{x\to -1}\dfrac{x^2+x-2}{x^2-1}$ (2 marks)

iii. $\lim\limits_{x\to 1}\dfrac{x^2+x-2}{x^2-x}$ (2 marks)

b)Differentiate the following functions with respect to x from first principles:

i. $y=2x+3$ (2 marks)

ii. $y=\dfrac{1}{x}$ (2 marks)

iii. $y=\sin x$ (4 marks)

c) Find the derivatives of the following:

i. $y = x^{-2}\left(4+3x^{-3}\right)$ (2 marks)

ii. $y = \sqrt{x^2+3x+2}$ (2 marks)

d)Find $\frac{dy}{dx}$ given that $y = \frac{2x+1}{3x^2+x+1}$ (3 marks)

e)Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given that :

i. $y = xe^{-\frac{x^2}{2}}$

ii. $y = \frac{Cosx}{x}$ (6 marks)

f)Find the equation of the tangent to the circle $x^2+4y^2=80$ at the point (1,1) (3 marks)

<u>QUESTION TWO (20MARKS)</u>

a)Find the gradient of the curves:

i) $x^2+2xy-2y^2+x=2$ at the point (-4,1) (4marks)

ii) $x = \frac{t}{1+t}, y = \frac{t^3}{1+t}$ at the point $(\frac{1}{2},\frac{1}{2})$ (4marks)

b) Find the $\lim_{x\to\infty}\sqrt{\frac{8x^2-3}{2x^2+x}}$ (2marks)

c) Find the value of $\frac{dy}{dx}$ at point (-1,1) of the function $x^2y+3y-4=0$ (3marks)

d) Show that if $f(x) = \frac{\sin x}{\cos x}$ *then* $f'(x) = \sec^2 x$ (3marks)

e)Investigate the stationary values of the function $f(x) = x^4-4x^3$ (4marks)

QUESTION THREE(20 MARKS)

a)A container in the shape of right circular cone of height 10cm and base radius 1cm is catching the drips from a tap leaking at 0.1 cm^3/s. find the rate at which the surface area is increasing when water is half way up the cone. (7marks))

b)i)Find the equations of the tangent and normal to the curve $y = x^3 + 8x + 5$ at $x = 2$

(4marks)

ii) Differentiate y=sin(2x+3) (2marks)

iii)Find $\frac{dy}{dx}$ if $y = x^2 e^x - 2xe^x + 2e^x$ (3Marks)

c)A farmer has 10,000m of fencing wire with much to fence three of his rectangular farm. The fourth side being an existing fence of his neighbour . Find in metres the dimension of the field of the largest possible area that can be enclosed . (4Marks)

QUESTION FOUR (20MARKS)

a)The equation of a curve is y = $3x^2$ - 4 x + 1

Find the gradient function of the curve and its value when x = 2 (2 marks)

(b) Determine:

(i) The equation of the tangent to the curve at the point (2, 5) (2 marks)

(ii) The angle which the tangent to the curves at the point (2, 5) makes with the horizontal (1 mark)

(iii) The equation of the line through the point (2, 5) which is perpendicular to the tangent in (b) (i) (2marks)

c)Write the equation of the tangent and normal of the equation $x^5 + y^5 - 2xy = 0$ at a point (1,1) (5Marks)

d)Differentiate:

i) $y = x^2 \ln x$ (3Marks)

ii) $y = \dfrac{e^x}{\sin x}$ (3Marks)

(iii) $y = \cos^2 x$ (2marks)

QUESTION FIVE (20MARKS)

a) By using chain rule find $\dfrac{dy}{dx}$ given that;

i) y=$(x^2 + 2x)^{-2}$ (3Marks)

ii)Use the second derivative` to determine whether the function y=$x^3 - 6x^2 + 9x + 2$ has a minimum or a maximum. (3Marks)

(ii)Find the equation of the tangent line to the circle $x^2 + y^2 = 25$ at a point (3,4) (4Marks)

b)(i)Find the coordinates of the points on the curve $y = x^3 - 2x + 7$ at which the gradient is 1.

(3Marks)

ii)Find the turning values of y on the graph $y = 5 + 24x - 9x^2 - 2x^3$ (4Marks)

c)The volume of a cube is increasing at the rate of 2cm^3 per second. Find the rate of change of the side of the base when its length is 3 cm. (3marks)