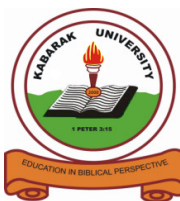


**KABARAK**



**UNIVERSITY**

**UNIVERSITY EXAMINATIONS  
MAIN CAMPUS**

**THIRD SEMESTER, 2015/2016 ACADEMIC YEAR**

**EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (SCIENCE)**

**MATH: 113 CALCULUS**

**STREAM: Y1S1**

**TIME: 3.00 – 5.00 PM**

**EXAMINATION SESSION: AUGUST**

**DATE: 11/8/2016**

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**INSTRUCTIONS:**

Answer question **one** and any other **two**

**QUESTION ONE (30MARKS)**

a) Evaluate the following limits:

i.  $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x^2 - 4}$  (2 marks)

ii.  $\lim_{x \rightarrow -1} \frac{x^2 + x - 2}{x^2 - 1}$  (2 marks)

iii.  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$  (2 marks)

b) Differentiate the following functions with respect to  $x$  from first principles:

i.  $y = 2x + 3$  (2 marks)

ii.  $y = \frac{1}{x}$  (2 marks)

iii.  $y = \sin x$  (4 marks)

c) Find the derivatives of the following:

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As members of Kabarak University family, we purpose at all times and in all places, to set apart in one's heart, Jesus as Lord. (1 Peter 3:15)

i.  $y = x^{-2}(4 + 3x^{-3})$  (2 marks)

ii.  $y = \sqrt{x^2 + 3x + 2}$  (2 marks)

d) Find  $\frac{dy}{dx}$  given that  $y = \frac{2x+1}{3x^2+x+1}$  (3 marks)

e) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  given that :

i.  $y = xe^{-\frac{x^2}{2}}$

ii.  $y = \frac{\cos x}{x}$  (6 marks)

f) Find the equation of the tangent to the circle  $x^2 + 4y^2 = 80$  at the point (1,1) (3 marks)

## **QUESTION TWO (20MARKS)**

a) Find the gradient of the curves:

i)  $x^2 + 2xy - 2y^2 + x = 2$  at the point (-4,1) (4marks)

ii)  $x = \frac{t}{1+t}, y = \frac{t^3}{1+t}$  at the point  $(\frac{1}{2}, \frac{1}{2})$  (4marks)

b) Find the  $\lim_{x \rightarrow \infty} \sqrt{\frac{8x^2 - 3}{2x^2 + x}}$  (2marks)

c) Find the value of  $\frac{dy}{dx}$  at point (-1,1) of the function  $x^2y + 3y - 4 = 0$  (3marks)

d) Show that if  $f(x) = \frac{\sin x}{\cos x}$  then  $f'(x) = \sec^2 x$  (3marks)

e) Investigate the stationary values of the function  $f(x) = x^4 - 4x^3$  (4marks)

**QUESTION THREE(20 MARKS)**

a)A container in the shape of right circular cone of height 10cm and base radius 1cm is catching the drips from a tap leaking at  $0.1 \text{ cm}^3/\text{s}$ . find the rate at which the surface area is increasing when water is half way up the cone. (7marks))

b)i)Find the equations of the tangent and normal to the curve  $y = x^3 + 8x + 5$  at  $x = 2$  (4marks)

ii) Differentiate  $y=\sin(2x+3)$  (2marks)

iii)Find  $\frac{dy}{dx}$  if  $y = x^2e^x - 2xe^x + 2e^x$  (3Marks)

c)A farmer has 10,000m of fencing wire with much to fence three of his rectangular farm. The fourth side being an existing fence of his neighbour . Find in metres the dimension of the field of the largest possible area that can be enclosed . (4Marks)

**QUESTION FOUR (20MARKS)**

a)The equation of a curve is  $y = 3x^2 - 4x + 1$

Find the gradient function of the curve and its value when  $x = 2$  (2 marks)

(b) Determine:

(i) The equation of the tangent to the curve at the point (2, 5) (2 marks)

(ii) The angle which the tangent to the curves at the point ( 2, 5) makes with the horizontal (1 mark)

(iii) The equation of the line through the point ( 2, 5) which is perpendicular to the tangent in (b) (i) (2marks)

c)Write the equation of the tangent and normal of the equation  $x^5 + y^5 - 2xy = 0$  at a point (1,1) (5Marks)

d)Differentiate:

i)  $y = x^2 \ln x$  (3Marks)

ii)  $y = \frac{e^x}{\sin x}$  (3Marks)

(iii)  $y = \cos^2 x$  (2marks)

**QUESTION FIVE (20MARKS)**

a) By using chain rule find  $\frac{dy}{dx}$  given that;

i)  $y = (x^2 + 2x)^{-2}$  (3Marks)

ii) Use the second derivative to determine whether the function  $y = x^3 - 6x^2 + 9x + 2$  has a minimum or a maximum. (3Marks)

(ii) Find the equation of the tangent line to the circle  $x^2 + y^2 = 25$  at a point (3,4) (4Marks)

b)(i) Find the coordinates of the points on the curve  $y = x^3 - 2x + 7$  at which the gradient is 1. (3Marks)

ii) Find the turning values of y on the graph  $y = 5 + 24x - 9x^2 - 2x^3$  (4Marks)

c) The volume of a cube is increasing at the rate of  $2\text{cm}^3$  per second. Find the rate of change of the side of the base when its length is 3 cm. (3marks)