KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2012/2013 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MATH 113: CALCULUS I

DAY: FRIDAY **DATE: 16/08/2013**

TIME: 10.00 - 12.00 P.M. **STREAM: Y1S1**

INSTRUCTIONS:

- Attempt **QUESTION ONE** and **ANY OTHER TWO** questions.

QUESTION ONE (30 MARKS)

a) (i) Given two elements a and b where a,b$\in \Re$, $\forall$ a<b, give four possible cases of intervals. (4 mks)

(ii) Using the inequality signs, represent the following intervals:

$(-\infty,+\infty)$, $(-\infty,a]$, $(a,+\infty)$, $(-\infty,a)$, $[a,+\infty)$. (4 mks)

b) Given Xn=$\frac{n}{2n+1}$ as $Xn \to \frac{1}{2}$, show that $\frac{1}{2}$ is the limit hence evaluate $N(\varepsilon)$ given $\in = 0.01$ (4 mks)

c) Evaluate the following limits

i) $\lim_{x\to\infty} \frac{x^2-2x+1}{2x^2+5}$ (2 mks)

ii) $\lim_{x\to 0} \frac{Cosx - Cos3x}{x^2}$ (2 mks)

iii) $\lim_{x\to\infty} \frac{(x-4)^{50}(2x+2)^{20}}{(6x^2-1)^{23}}$ (2 mks)

iv) $\lim_{x\to\infty}\{3^x+3^{2x}\}^{\frac{1}{x}}$ (2mks)

d) Derive the quotient rule of differentiation and hence differentiate the function $y=\dfrac{6x^2+x}{5x^2}$ (5mks)

e) Using the first principle technique differentiate y = e^{2x}

(3 mks)

f) Given $y=\dfrac{1}{x}$ and x>0, use a sketch to show the two limiting values of the sequence.

(2 mks)

QUESTION TWO (20 MARKS)

(a) Show that:

(i) $\dfrac{d}{dx}$ Sin x = Cos x (4mks)

(ii) $\dfrac{d}{dx}$ Cos x = -Sin x (4mks)

(b) Differentiate the following functions w.r.t x

(i) y = $\dfrac{e^{-ax}+e^{ax}}{e^{ax}}$ (4mks)

(ii) y = $\text{Cos}^2(4x^2)+\text{Sin}^3 2x$ (3mks)

(c) Evaluate the following Limit

$\lim_{x\to-\infty}\left(1+\dfrac{3}{x}\right)^{x+4}$ (2mks)

d) Using $\in-\delta$ definition of a limit verify the following Limit.

$\lim_{x\to 2}(x^3+x+1)=11$ (3mks)

QUESTION THREE (20 MARKS)

a) With a good illustration deduce the squeeze principle of limits and hence evaluate $\lim_{x\to\infty}\dfrac{\cos^2 2x}{3-2x}$ (8mks)

b) Find $\dfrac{dy}{dx}$ of the following functions at a specified points

i. i) $y=\dfrac{(2x^2+3x+2)^2}{(x+3)^2}$ at x=0 (3 mks)

ii. $y=\sqrt{x^2+2x}$ at x=1 (3 mks)

iii. $y=(2x^2+x+4)(x^2+2x+1)^2$ at x=0 (3 mks)

iv. $y=\ln^{2(}x^2+4x+15)^3$ at x=10 (3mks)

QUESTION FOUR (20 MARKS)

a) Determine whether $y=Ae^{ax}+Be^{-ax}$ is satisfied by $y^{11}-a^2y=0$. (2 mks)

(b) Write the equation of the tangent and normal to the curve:
$x^5+y^5-2xy=0$ at a point (1,1). (4 mks)

(c) Investigate the local extrema of the function.
$f(x) = 2x^3-3x^2 - 12x + 5$ (3mks)

(d) The gradient of a curve is 4x – 2. Find the equation of the curve given x – axis is a tangent to the curve. (3mks)

(e) Find the radius of a cylindrical tank given that the volume is 16π (mls) if the area have to be least. (4mks)

(f) A ball was thrown upwards with a velocity of 40m/s; find the acceleration, velocity and distance expression, hence find the maximum height the ball can attain. (4mks)

QUESTION FIVE (20 MARKS)

(a) Let D be the region in the plane bounded by the graph of f(x)=x and $g(x)=x^2$
Find the volume of the solid formed by resolving D about the x-axis. (5mks)

(b) Find the maclaurin series of expansion $(1+x)^{\frac{1}{2}}$. (5mks)

(c) If the derivative of a function is 0 throughout an interval, then show that the function is constant throughout the interval. (6mks)

(d) Determine whether f(x)=3x+2sinx is increasing or decreasing or neither. (4mks)