KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2013 / 2014 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MATH 113: CALCULUS (58 STUDENTS)

DAY: **DATE:**

TIME: 2Hrs **STREAM: Y1S1**

INSTRUCTIONS:
Answer Question One And Any Other Two Questions

QUESTION ONE – (30 MARKS)

a) State the $\varepsilon - \delta$ definition of a limit of a function $f(x)$ as x approaches a point $x = a$ and use it to prove that $lim_{x \to 2}(2x - 1) = 3$ (4 marks)

b) Let $f(x) = \begin{cases} x^2 & x < 1 \\ 4 & x = 1 \\ 2x - 1 & x > 1 \end{cases}$

(i) Graph f

(ii) Evaluate $\lim_{x \to 1^-} f(x)$ and $lim_{x \to 1^+} f(x)$

(iii) Is the function continuous at $x = 1$? Explain. (6 marks)

c) Use the first principle method to differentiate $f(x) = \sqrt{2x + 1}$ with respect to x.
(5 marks)

d) Find $\frac{dy}{dx}$ if:

i) $y = (\sin 3x)^{\cos 3x}$ (3 marks)

ii) $y = \frac{1+ e^{x^2}}{1- e^{3x}}$ (3 marks)

iii) $y = tan^2(2x - x^3)$ (3 marks)

e) Find the equation of the tangent to the curve $x^2 + y^2 = y$ at point $(1, 1)$ (3 marks)

f) Deduce product rule of differentiation (3 marks)

QUESTION TWO – (20 MARKS)

a) Determine whether $y=Ae^{ax}+Be^{-ax}$ is satisfied by $y^{11}-a^2y=0$. (2 marks)

(b) Write the equation of the tangent and normal to the curve:
$x^5+y^5-2xy=0$ at a point (1,1). (4 marks)

(c) Investigate the local extrema of the function.
$f(x) = 2x^3-3x^2 – 12x + 5$ (3marks)

(d) The gradient of a curve is 4x – 2. Find the equation of the curve given x – axis is a tangent to the curve. (3marks)

(e) Find the radius of a cylindrical tank given that the volume is 16π (mls) if the area have to be least. (4marks)

(f) A ball was thrown upwards with a velocity of 40m/s; find the acceleration ,velocity and distance expression, hence find the maximum height the ball can attain. (4marks)

QUESTION THREE – (20 MARKS)

a) Show that the function $f(x) = \begin{cases} 2x + 3, & x < 0 \\ x + 3, & x \geq 0 \end{cases}$ is not differentiable at $x = 0$

(5 marks)

b) Given that $y = \frac{1}{1-x}$ show that:

(i) $\frac{dy}{dx} = y^2$ (1 mk)

(ii) $\frac{d^2y}{dx^2} = y^3$ (2 marks)

(iii) $\frac{d^3y}{dx^3} = y^4$ (2 marks)

(iv) Hence find the constant a and b such that $\frac{d^{20}y}{dy^{20}} = \frac{a}{y^b}$ (3 marks)

c) If $y = \frac{\cos x}{x}$, prove that $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + y = 0$ (7 marks)

QUESTION FOUR – (20 MARKS)

a) Find the expression for $\frac{dy}{dx}$ given that

(i) $y = 3x^7 - 7x^3 + 4x - 21$ (2 marks)

(ii) $y = In\left(\sqrt{x^2+1-x}\right)$ (2 marks)

(iii) $y = \frac{\sin x}{1-\cos x}$ (4 marks)

(iv) $y = tan^{-1}\left(\frac{x+1}{x-1}\right)$ (5 marks)

b) By differentiating implicitly find $\frac{dy}{dx}$ given that

(i) $y^3 - 3x^2y + 2x^3 = 0$ (4 marks)

(ii) $In\ xy = e^{x+y}$ (3 marks)

QUESTION FIVE – (20 MARKS)

a) Let $f(x) = -2x^4 + 6x^2 - 3$. Find the extreme values of f, and sketch the graph of f. (6 marks)

b) A crate open at the top has vertical sides, a square bottom, and a volume of 4 cubic meters. If the crate has the least possible surface area, find its dimensions. (5 marks)

c) Use L'Hospital's rule to find

(i) $lim_{x\to\infty} \frac{In\ x}{x}$ (2 marks)

(ii) $lim_{x\to 1} \frac{x^3-1}{4x^3-x-3}$ (3 marks)

(iii) $lim_{x\to 0}\left(\frac{e^x-e^{-x}-2x}{x-\sin x}\right)$ (4 marks)