

**KABARAK**



**UNIVERSITY**

**EXAMINATIONS**

**2008/2009 ACADEMIC YEAR**

**FOR THE DEGREE OF BACHELOR OF EDUCATION  
SCIENCE**

**COURSE CODE: MATH 121**

**COURSE TITLE: CALCULUS II**

**STREAM: SESSION II & III**

**DAY: THURSDAY**

**TIME: 2.00 – 4.00 P.M.**

**DATE: 09/04/2009**

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**INSTRUCTIONS:**

Answer question **ONE** and any other **TWO**

PLEASE TURN OVER

QUESTION ONE (30 MARKS)

(a) Evaluate the following integrals

(i)  $\int x \sin(x^2) dx$  (3 marks)

(ii)  $\int \ln x dx$  (4 marks)

(iii)  $\int \frac{x+2}{1-x} dx$  (3 marks)

(b) Find the solution to the differential equation

$$(1+x^2) \frac{dy}{dx} + xe^{-y} = 0, \text{ given that } x=0, y=1. \quad (5 \text{ marks})$$

(c) Find the area between the curve  $y^2 = x$  and the line  $x = 2$  (4 marks)

(d) Use the trapezoidal rule with  $n=4$  to estimate  $\int_1^2 x^2 dx$  and calculate the difference between the estimate and the exact answer. (5 marks)

(e) Find the length of the curve

$$\frac{1}{3}(x^2 + 2)^{\frac{3}{2}} \text{ between } x=0 \text{ and } x=3. \quad (6 \text{ marks})$$

QUESTION TWO (20 MARKS)

(a) Integrate by parts with respect to  $x$

$$x^2 \sin x \quad (5 \text{ marks})$$

(b) Use partial fractions to evaluate  $\int_3^5 \frac{2}{x^2-1} dx$  (7 marks)

(c) Use the substitution methods to solve

(i)  $\int \frac{dx}{\sqrt{4-x^2}}$  (4 marks)

(ii)  $\int_0^1 x(1+x^2)^{\frac{3}{2}} dx$  (4 marks)

QUESTION THREE (20 MARKS)

(a) Resolve  $\frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2}$  into partial fractions. Hence evaluate

$$\int \frac{x^3 + x^2 + x + 2}{x^4 + 3x^2 + 2} dx \quad [\text{Hint: } x^2 + 1 \text{ is a factor of } x^4 + 3x^2 + 2]. \quad (10 \text{ marks})$$

(b) Evaluate  $\int_1^2 \frac{dx}{x^2 \sqrt{x-1}}$  by means of substitution  $x = \sec^2 \theta$ . (10 marks)

QUESTION FOUR (20 MARKS)

(a) If  $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$ , Prove that for  $n \geq 2$

$$I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}. \quad \text{Hence calculate the value of}$$

$$\int_0^{\frac{\pi}{2}} x^5 \sin x dx. \quad (14 \text{ marks})$$

(b) Evaluate  $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^3 x dx$ . (6 marks)

QUESTION FIVE (20 MARKS)

(a) Find the volume generated when the area enclosed by the x-axis and the curve

$$y = 3x^2 - x^3 \text{ is rotated about x-axis.} \quad (4 \text{ marks})$$

(b) Tabulate to 3 decimal places the values of the function  $f(x) = \frac{1}{x}$  for values of  $x$

from 1 to 2 at intervals of 0.1. (4 marks)

(c) Use the values in (a) above to estimate  $\int_0^1 \frac{1}{x} dx$  by

(i) Trapezoidal rule (ii) Simpson's rule (8 marks)

(c) Evaluate  $\int_1^2 \frac{1}{x} dx$  by integration. Hence determine the percentage error in the values

obtained in (b) above. (4 marks)