KABARAK **UNIVERSITY**

UNIVERSITY EXAMINATIONS

2009/2010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: **MATH 121**

COURSE TITLE: **INTEGRAL CALCULUS**

STREAM: **Y1S2**

DAY: **THURSDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **18/03/2010**

INSTRUCTIONS:

1. Question ONE is compulsory.
2. Attempt question ONE and any other TWO

PLEASE TURN OVER

Question One   [30 Marks]

a) Find the most general antiderivative of the function $f(x) = (4 + 3x^2 \cos 4x)/x^2$

[3 marks]

b) Use the method of substitution to evaluate $\int_{2}^{10} \dfrac{3}{\sqrt{5x-1}}\,dx$ [4 marks]

c) Find the area of the region bounded by the graphs of the equations $y + x^2 = 6$ and

$y + 2x - 3 = 0$ [4 marks]

d) Given that $y'' = x^2 - 6$, $y'(0) = 2$ and $y(1) = -1$ find y.   [4 marks]

e) Use trapezoidal rule with n = 4 to approximate the value of $\int_{0}^{\pi} x^2 \sin x\,dx$ [4 marks]

f) Find y' if $\sinh xy = ye^x$ [4 marks]

g) Evaluate $\int \ln x\,dx$ [3 marks]

h) The integral $\int_{2}^{\infty} \dfrac{1}{(x-1)^2}$ converges find its value.   [4 marks]

Question Two   [20 Marks]

a) If n is a rational and $n \neq -1$ show that   [5 marks]

$$\int u'(x)[u(x)]^n\,dx = \frac{[u(x)]^{n+1}}{n+1} + c$$

b) Use differentials to approximate the arc length of the graph of $y = x^3 + 2x$ from

$A(1,3)$ to $B(1.2, 4.128)$ [5 marks]

c) A stone is thrown directly downward from a height of 96 feet with an initial velocity

16 ft/sec. Find

    i)   its distance above the ground after t seconds   [3 marks]

    ii)   when it will strike the ground   [3 marks]

    iii)   the velocity at which it strikes the ground.   [4 marks]

Question Three   [20 Marks]

a) Show that $\int \dfrac{e}{16-e^{2x}} dx = \dfrac{1}{4}\tanh^{-1}\dfrac{e^{x}}{4} + c$ [6 marks]

b) Evaluate $\int \dfrac{x^{2}-x-21}{2x^{3}-x^{2}+8x-4} dx$ [8 marks]

c) To monitor the thermal pollution of a river, a biologist takes hourly temperature readings (in ^{o}F) from 9 A. M to 5 P. M. the results are shown in the table below. Use Simpson's rule to estimate the average water temperature between 9 A. M to 5 P. M.

           [6 marks]

Time of day	9	10	11	12	1	2	3	4	5
Temperature	75.3	77.0	83.1	84.8	86.5	86.4	81.1	78.6	75.1

Question Four [20 Marks]

a) Use integration by parts to evaluate $\int_{1}^{2} x^{5}e^{x^{3}} dx$ [7 marks]

b) The region bounded by the $y-axis$ and the graphs of $y = x^{3}$, $y = 1$ and $y = 8$ is revolved about the $y-axis$. Find the volume of the resulting solid.   [8 marks]

c) Evaluate $\int_{0}^{2}\int_{x^{2}}^{2x}\left(x^{3}+4y\right) dydx$ [5 marks]

Question Five [20 Marks]

a) Evaluate $\int \cos^{3}x\sin^{4}x dx$ [7 marks]

b) Use trigonometric substitution to evaluate $\int \dfrac{\sqrt{x^{2}-9}}{x} dx$ [7 marks]

c) Find the second partial derivatives of f if $f(x, y) = x^{3}y^{2} - 2x^{2}y + 3x$ [6 marks]