KABARAK   **UNIVERSITY**

UNIVERSITY EXAMINATIONS

2014 / 2015 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

MATH 322: ORDINARY DIFFERENTIAL EQUATIONS II

DAY: SATURDAY   **DATE: 28/11/2015**

TIME: 11.00 –1.00 P.M   **STREAM: Y3S2**

INSTRUCTIONS:

Answer Question ONE and any other two.

QUESTION ONE (30 Marks)

     a) Define the terms
           i.   regular point
           ii.   wronskian

     b) Find the equation of the tangent and normal to the circle $x^2 + y^2 = 8$ at the point $M_O(2,2)$ (6 marks)

     c) Find the general solution of $\dot{x}_1 = x_1 + x_2, \dot{x}_2 = x_1 + x_2,$ (8 marks)

     d) Determine a fundamental matrix for $x' = At$ where A=$\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ (8 marks)

     e) Express the initial value problem (IVP) $\ddot{x} + 2\dot{x} - 8x = 0 ; x(1) = 2 , \dot{x} = 3$ in the form $\dot{x}(t) = A(t)x(t) + f(t)$ (8 marks)

QUESTION TWO (20 Marks)

     a) Reduce the equation $x^2y'' - 3xy' + 4y = 0$ to first order linear form   (6marks)

     b) Given $\frac{dy}{dx} = 3x^{\frac{2}{3}}$, at $x(0) = 0$. Determine whether $f(x,t)$ satisfy Lipschyt's condition near (1,1).   (4marks)

     c) Find the wronskian determinant $\omega(t)$ of the functions

    i. e^t, cos t , sin t,

    ii. $e^t, e^{-t}, e^{it}, e^{-it}$ (6marks)

(d) Show that $y' = \frac{1}{x}$ with, y(0) = 0 has no solution   (4marks)

QUESTION THREE (20 marks)

(a) Given the set $\{sin\ x\ , cos\ x, e^{ix}\}$find the wronskian, giving reasons determine whether the functions are dependent or independent   (8marks)

(b) Determine whether $x\frac{dy}{dx}=2y$, has a unique solution   (4 marks)

(c) Find the power series solution of the equation

$$x\frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 1 \qquad\qquad (8\text{marks})$$

QUESTION FOUR (20 marks)

a) Find the general solution near $x = 0$ of $3x^2 y'' - xy' + y = 0$ (8marks)

b) For each of following equation determine which points are Ordinary, Regular, Neither

    i. $x(1 + x)y'' - (a + 4x)y'' + 2y = 0$ Where is a real constant   (6marks)

    ii. $x^3 y'' - xy' - y = 0$ (6marks)

QUESTION FIVE (20 marks)

a) Given the set $\{x, x^2\}$ determine the Wronskian$W(x)$ hence show that the functions are independent.   (6marks)

b) Find e^{At} for A= $\begin{bmatrix} 1 & 1 \\ 9 & 1 \end{bmatrix}$ (8marks)

c) Determine a series solution of the equation

    $\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$ (6marks)

d) Solve $x'' + 4x = 8t^2 - 4t + 1$ (4marks)