KENYATTA UNIVERSITY

UNIVERSITY EXAMINATIONS 2017/2018
FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ECONOMICS (ECONOMICS & FINANCE), BACHELOR OF ECONOMICS AND STATISTICS AND BACHELOR OF ARTS

EES 200: MATHEMATICS FOR ECONOMISTS II

DATE: Friday, 2nd February, 2018 TIME: 11.00 a.m. - 1.00 p.m.

INSTRUCTIONS: Answer question **ONE** and any other **TWO** questions.

QUESTION ONE

a) Suppose the value of timber is given by $2^{\sqrt{t}}$ where t is the number of years. Assume a discount rate of $r = 0.05$ and also assume zero upkeep cost during the period of timber growth. What is the optimal time to cut the timber for sale? **(5 Marks)**

b) Find the extreme value of $z = -3x^2 + 6x + xy + 46 - y^2$ and determine whether the extreme point is a maximum or a minimum. **(5 Marks)**

c) Find dy/dx for the following function

$$2x^2y^4 - xy^5 + x^3 + 3y^2 = 20$$ **(5 Marks)**

d) What is the rank of the following matrix $\begin{bmatrix} 6 & 3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix}$

(5 Marks)

e) Use natural logarithms to get dy/dx given $y = \dfrac{3x}{(x+2)(x+4)}$ **(4 Marks)**

f) Evaluate

i) $\int 3e^{-(2x+7)}dx$

ii) $\int (5e^x + \frac{3}{x^2})dx$

iii) $\int x \ln x dx$ **(6 Marks)**

INVOLVEMENT IN ANY EXAMINATION IRREGULARITY SHALL LEAD TO DISCONTINUATION

QUESTION TWO

a) Find the critical value of

$z = 2x^2 - 2xy + 4y^2$

$s.t.$

$2x + 2y - 12 = 0$

and determine whether the critical value is a maximum or minimum. (6 Marks)

b) Find dz and dz^2 for the function $z = x^2 + xy + 2y^2 + 3$ (3 Marks)

c) A firm has a production function of form $y = 500 - x_1^2 - x_2^2$, where x_1 stands for labour and x_2 stands for capital. The cost of labour is given as KES300 per unit and the cost of capital is given as KES600 per unit. The firm wishes to maximize its output for a cost of KES64000.

i) What are the levels of x_1 and x_2 for which y is maximized? (6 Marks)

ii) Is the second order condition satisfied? (3 Marks)

iii) What is the maximum y? (2 Marks)

QUESTION THREE

a) Solve the following system of linear simultaneous equations using Cramer's rule

$8x_1 - x_2 = 16$

$2x_2 + 5x_3 = 5$ (6 Marks)

$2x_1 + 3x_3 = 7$

b) Given $u = \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$ and $v = \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}$, evaluate uv' and $d(u, v)$ (4 Marks)

c) You are given the input matrix and the final demand vector as

$$A = \begin{bmatrix} 0.05 & 0.25 & 0.34 \\ 0.33 & 0.10 & 0.12 \\ 0.19 & 0.38 & 0 \end{bmatrix}, d = \begin{bmatrix} 1800 \\ 200 \\ 900 \end{bmatrix}$$

i) Write the specific input-output matrix equation for this model (2 Marks)

ii) Hence solve for the required output. (8 Marks)

QUESTION FOUR

a) Evaluate the derivatives of

i) $y = \ln t^c$

ii) $y = t^3 \ln t^2$

(5 Marks)

b) Find the differential dy of $y = 3x^4 + 5x^3 - 4x^2 + 7$ and find the change in y given that x changes from 1 to 1.001.

(5 Marks)

c) Assume that the rate of investment is described by the function $I(t) = 12t^{\frac{1}{3}}$ and that $K(0) = 25$.

i) Find the time path of the capital stock k

(2 Marks)

ii) Find the amount of capital accumulation during the time interval (0, 1) and (1, 3) respectively.

(3 Marks)

d) You given the demand function as $P = 14 - 2Q - Q_2$. Determine the consumer surplus at $P = 4$.

(5 Marks)

QUESTION FIVE

a) You are given the following matrices

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 2 & 4 \\ 5 & 7 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 3 & 5 \\ 8 & 9 & 6 \end{bmatrix}$$

i) In which order are two matrices conformable with respect to multiplication? (1 mark)

ii) Get the product the from your answer in (i)

(3 Marks)

b) Given a two-commodity market model represented by

$$Q_{d1} = 4 - P_1 + \frac{1}{2}P_2 \qquad Q_{d2} = 8 + 2P_1 - 2P_2$$

and

$$Q_{S1} = -\frac{5}{2} + \frac{3}{2}P_1 \qquad Q_{S2} = -3 + 6P_2$$

Calculate the equilibrium prices and quantities for the market using matrix inversion method.

(8 Marks)

c) Demonstrate Eulers Theorem given the following function

$Q = 5K^{2/5}L^{8/5}$

(4 Marks)

d) You are given $MR(Q) = 28Q - e^{0.3Q}$.

i) Get the total revenue (TR) function (2 Marks

ii) What initial condition can you introduce to definitize the constant? (2 Marks)