DEE VOL 103

Name:_______________ Index No.:_______________

2201/301
2203/301
2204/301
2206/301
MATHEMATICS
Oct./Nov. 2014
Time: 3 hours

Candidate's Signature:_______________

Date:_______________

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN ELECTRONICS ENGINEERING
DIPLOMA IN TELECOMMUNICATION ENGINEERING
DIPLOMA IN ELECTRICAL ENGINEERING (POWER OPTION)
DIPLOMA IN INSTRUMENTATION AND CONTROL ENGINEERING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

Write your name and index number in the spaces provided above.
Sign and write the date of the examination in the spaces provided above.
You should have the following for this examination:
Mathematical tables/Scientific calculator;
Geometrical drawing instruments.
An abridged table of Laplace Transforms and the Standard Normal Tables are attached.
Answer any ***FIVE*** *of the following* ***EIGHT*** *questions in the spaces provided in this question paper.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Do ***NOT*** *remove any pages from this booklet.*
Candidates should answer the questions in English.

For Examiner's Use Only

Question	1	2	3	4	5	6	7	8	TOTAL MARKS
Candidate's Score									

This paper consists of 24 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Find the eigen values and corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$$

(10 marks)

(b) A linear time-invarient system is characterized by the vector differential equation

$$\frac{d\underset{\sim}{x}}{dt} = A\underset{\sim}{x}\text{, where } A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Find the state transition matrix Φt, as the system. (10 marks)

2. (a) A function f (x) is defined by

$$f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2 - x), & 0 < x < 1 \end{cases}$$

Obtain the half-range cosine series of the function f(x) and hence show that

$$\frac{\pi^4}{96} = \sum_{n=1}^{\alpha} \frac{1}{(2n - 1)^4}$$

(20 marks)

3. (a) Use Green's theorem in a plane to evaluate the integral

$$\oint_c [(2x^2 - y^2)dx + (x^2 + y^2)dy] \text{ where}$$

c is the boundary in the xy plane of the area enclosed by the x - axis and the semi-circle $x^2 + y^2 = 2$ in the upper half of xy plane. (9 marks)

(b) A solid is bounded by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + 3y^2$. Determine the volume of the solid. (11 marks)

4. A continuous random variable x has a probability density function f(x) given by

$$f(x)=\begin{cases} ke^{-2x} & \text{for } x>0 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Determine the:

(a) value of the constant k ;
(b) mean and variance of x ;
(c) median of the distribution;
(d) $p(x>1)$

(20 marks)

5. (a) Find the general solution of the differential equation

$$(2x+3\cos y)dx+(2y-3x\sin y)dy=0$$

(9 marks)

(b) Use the method of undetermined coefficients to solve the differential equation

$$\frac{dy^2}{dx^2}+2\frac{dy}{dx}+10y=\sin 3x$$

given that $y=0$ and $\frac{dy}{dx}=1$ when $x=0$

(11 marks)

6. (a) Find the inverse laplace transform of

$$F(s)=\frac{s+4}{s(s+1)(s^2+4)}$$

(8 marks)

(b) A unit e.m.f. is applied to the terminals 'A' and 'B' of the network in the **figure 1** below, which is dead at $t = 0$. Use laplace transforms to show that the input current $i(t)$ is given by

$$i(t) = 2\left\{1 - e^{\frac{-3}{2}t}\cosh\frac{\sqrt{5}}{2}t - \frac{2}{\sqrt{5}}e^{\frac{-3}{2}t}\sinh\frac{\sqrt{5}}{2}t\right\}$$

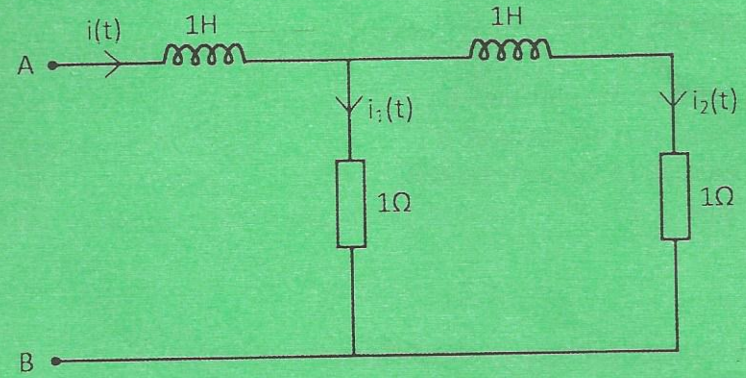


Fig. 1

(12 marks)

7. (a) Show that the real and imaginary parts of the function $w = \ln z$ satisfy the Canchy-Riemann equations. (10 marks)

(b) Given that $|z| = 2$ and

$$w = f(z) = j\left(\frac{1 - z}{1 + z}\right),$$

Find the image of z in the w-plane. (10 marks)

8. (a) Prove that for any real number N, the Newton-Raphson formula for approximating $N^{\frac{1}{3}}$ can be written in the form

$$x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{X_n^2}\right)$$

Hence evaluate $3^{\frac{1}{3}}$ correct to four decimal places, giving that an approximate root is 1. (9 marks)

(b) The table below represents a polynomial f(x) and an error is suspected in one of the entries.

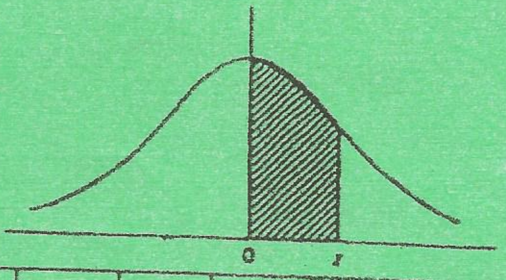
x	0	1	2	3	4	5	6	7	8	9	10
f(x)	5	9	37	101	213	395	629	957	1381	1913	2565

(i) Use finite differences to locate and correct the error;

(ii) Use Gregory-Newton interpolation formula to determine f(6.8) correct to three decimal places;

(iii) Determine the form of the function f(x). (11 marks)

Partial areas under the standardised normal curve



$z = \frac{x - \bar{x}}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0159	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0678	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1388	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1891	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2086	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2760	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3215	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3451	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4430	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4762	0.4767
2.0	0.4772	0.4778	0.4783	0.4785	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4882	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4980	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\, t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$