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**University Examinations 2015/2016**

THRD YEAR, FIRT SEMESTER EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE ACTUARIAL SCIENCE AND BACHELOR OF SCIENCE (PHYSICAL OPTION)

**STA 2306/SMA 2301; REAL ANALYSIS FOR STATISTICS/ REAL ANALYSIS I**

**DATE: November, 2015 TIME: HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE – (30 MARKS)**

1. (i) Prove that despite the fact that there exists other number on the real number line which are not rational, between any two rational numbers there is another rational number.

 (2 Marks)

(ii) Show that (3 Marks)

1. For any two sets x and y, show that

 (3 Marks)

1. Show that there is no rational number whose square is 5. (4 Marks)
2. Use the comparison test to determine the convergence of the series

(3 Marks)

1. Consider the set show that Sup S = 2 (3 Marks)
2. Use the first principle to show that;

 (3 Marks)

1. Let  and  where and are sequences of real numbers. Then show that the  (4 Marks)
2. Show that a function which is uniformly continuous on an interval is continuous on that interval. (3 Marks)
3. Calculate the limit superior and limit inferior on the sequence ; n

(2 Marks)

**QUESTION TWO (20 MARKS)**

1. (i) State the Cantor’s theorem on function without proof. (2 Marks)

(ii) Show that is uniformly continuous on the interval (0,2) but not uniformly continuous on the interval (0,). (4 Marks)

1. Show that a function is not continuous on the interval (0,1) (4 Marks)
2. Consider the set for all x. x = (). Define d : x

 by d(x,y) = max (i =1,2, …,n) Prove that ( ,d) is a metric space.

(5 Marks)

1. Show that the sequence is a Cauchy sequence. (5 Marks)

**QUESTION THREE (20 MARKS)**

1. Let be a sequence of real numbers, prove that if converges, then its limit is unique. (5 Marks)
2. Prove that a sequence of real numbers converges to a limit x if and only if every subsequence of converges to x. (5 Marks)
3. Prove that every convergent sequence is bounded and with the help of an example, show that the converge is not necessarily true. (9 Marks)

**QUESTION FOUR (20 MARKS)**

1. Use the Cauchy’s integral test to show that ;

  (i) Converges if P > 1

 (ii) Diverges if P (6 Marks)

1. Prove that set of real numbers is uncountable. (5 Marks)
2. Suppose that  and  are positive series if k = 1, 2, 3, 000 Show that  will converge if converges. (4 Marks)
3. Let M and N be neighbourhood of point x. Show that MN is also a neighbourhood of x

 (5 Marks)

**QUESTION FIVE (20 MARKS)**

1. (i) Consider three sets . Show that Z (3 Marks)

(ii) Hence prove that for =(x\) (4 Marks)

1. Show that the sets (x\y) and xY) are disjoint and that their union is X. (6 Marks)
2. Prove that in any space , the finite intersection of a family of open sets is open.(4 Marks)
3. Let S be a non-empty subset of IR. Prove that the real no A is the supremum Iff both the following conditions are satisfied.
4. x
5. (3 Marks)