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**University Examinations 2015/2016**

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR DEGREE OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

**SMA 2401; TOPOLOGY I**

**DATE: November, 2015 TIME: HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE – (30 MARKS)**

1. Distinguish the following types of topologies. (4 Marks)
2. Discrete topology
3. Indiscrete topology
4. Sierpinskis topology
5. Euclidean topology
6. x= and = show that is a topology on x. (3 Marks)
7. Let f: X and g: YZ be continuous functios. Show that the composite g 0 f is also continuous. (4 Marks)
8. Let (x , be topological space. Denote a derived set of A by . Prove that a subset AX is closed iff A (5 Marks)
9. (i) Define a T4 –space. (1 Mark)

(ii) Let X be a T4-space , Then show that x also a regular space. (3 Marks)

1. Show that any class D of subsets of x is a subbase for a unique topology on X, ie finite intersection of members of D form the base for the topology on x (4 Marks)
2. Let (x , ) be a topological space. Prove that a subset AX is open iff its complement x|A is closed. (3 Marks)
3. Let be bases respectively for topologies . Suppose each open set Bis the onion of members of Show that is coarser that . (3 Marks)

**QUESTION TWO (20 MARKS)**

1. (i) Define a limit point of a subset A of a topological space x. (1 Mark)

(ii) Let x = and =

Let a = . Show that b is a limit point of A but a is not. (5 Marks)

(iii) Show that if x is a discrete topological space and that AX, then the derived set of A, (4 Marks)

1. Let (x,) be a topological space and A,BX. Prove that = (3 Marks)
2. Let (x,) be a topological space and A,B X. Denote the interior of A.
3. Using an appropriate example show that;

U (3 Marks)

1. prove that (4 Marks

**QUESTION THREE (20 MARKS)**

1. Let PEX and denote Np the set of all neighbourhood or point p. Prove that the following;
2. (3 Marks)
3. If NNp and for every MX with NM it implies that MNp. (3 Marks)
4. Consider the following topology on X = {a,b,c,d,e} and

= If A= Find;

1. The exterior of A (3 Marks)
2. The boundary of A (3 Marks)
3. Hence show that the boundary of A(A) = (3 Marks)
4. Show that the property of regular space is hereditary. (5 Marks)

**QUESTION FOUR (20 MARKS)**

1. Let with ={ and let y = with

=

Define the function F as



Show that the function F is a homomorphism. (5 Marks)

1. Let (x, ) and () be topological spaces, prove that F: Xis closed if and only if

F(A) F() A X (5 Marks)

1. Prove that a topological space x is a space iff every singleton subset X is closed. (5 Marks)
2. Show that every point P in a discrete topological space has a finite local base.(4 Marks)

**QUESTION FIVE (20 MARKS)**

1. Let F: be a objective. Prove that the following statements are equivalent;(10 Marks)
2. F is a homomorphism
3. F is open
4. F is closed
5. F() =
6. Let P: be a closed map. let S Y be any subset of Y and U X be any open set in X such that P-1(s) U, prove that there exists an open set V Y such that S V and

V (4 Marks)

1. Let for the topologies and on x. Prove that (6 Marks)