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**University Examinations 2015/2016**

FOURTH YEAR, FIRST SEMESTER EXAMINATION FOR DEGREE OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

**SMA 2407 ; FUNCTIONAL ANALYSIS**

**DATE: November, 2015 TIME:** $2$**HOURS**

**INSTRUCTIONS:** *Answer question* ***one*** *and any other* ***two*** *questions*

**QUESTION ONE – (30 MARKS)**

1. (i) Define a metric space . (3 Marks)

(ii) Give four examples of metric spaces. (4 Marks)

1. Show that d(x, y) = $(x – y)^{2}$ defines a metric on the set of all real numbers. (4 Marks)
2. Prove that T :$ R^{2}\rightarrow R^{2}$ defined by T(x ,y) = (2x, x+y) for all x,y $ϵ R^{2}$ is a linear operator. (5 Marks)
3. If in a inner product space $x\_{n}\rightarrow x$ and $y\_{n}\rightarrow y$ prove that $<x\_{n},y\_{n}> \rightarrow <x, y>$

(5 Marks)

1. Given the linear operator defined by the matrix.

A = $\left[\begin{matrix}1&2&3\\0&-1&1\\1&1&4\end{matrix}\right]$ Determine

1. The kernel of A (5 Marks)
2. The range of A (4 Marks)

**QUESTION TWO ( 20 MARKS)**

1. An inner product on X is a mapping XxX into scalar field K of X; state the properties of the inner product. (4 Marks)
2. Define orthogonality of two elements x and y in an inner product space X.(3 Marks)
3. The space $R^{n}$ is a Hilbert space with inner product defined by

$<x ,y> = x\_{1}y\_{1}$+$x\_{2}y\_{2}$+ … + $x\_{n}y\_{2}$+ … + $x\_{n}y\_{n}$ where x = ($x\_{1, }x\_{2 }$….,$x\_{n}$) and

 y = $\left(y\_{1, }y\_{2, }…y\_{n}\right)$

1. $If n=3, what is the inner$ product of $R^{3}$ (3 Marks)
2. Given that x = (1, 2,3) and y = (2, -1, 0) show that x and y are orthogonal by use of inner product of two vectors. Give the inner product of each of x and y

(10 Marks)

**QUESTION THREE (20 MARKS)**

1. Define a discrete metric space. (3 Marks)
2. State the Euclidean metric on $R^{2}$ and illustrate the same on the plane. (4 Marks)
3. (i) Let x be the set of ordered triples of zeros and ones. Show that x consists of eight elements and list them. (4 Marks)

(ii) Give the definition of the metric d on x. What is the name of d(x,y), the distance between x and y. (4 Marks)

1. In $R^{n}$ the norm of x is given by . State the norm when n=3 and conclude that it I the same as the notion of length $\left|x\right|$ of a vector in $R^{3}$ (5 Marks)

**QUESTION FOUR( 20 MARKS)**

1. State the properties of a norm. (4 Marks)
2. There are several norms of practical importance on the vector space $R^{n}$ of ordered n-triples

of numbers, for example those defined by:

1. $\left‖x\right‖ \_{1}$ = $\left|x\_{1}\right|$ +$\left|x\_{2}\right|$ +…..+ $\left|x\_{n}\right|$
2. $\left‖x\right‖\_{p}= \left(\left|x\_{1}\right|^{p}+\left|x\_{2}\right|^{p}+… + \left|x\_{n}\right|^{p}\right) \frac{1}{p} \left(l <p<\infty \right)$ in each case verify that the four properties of a norm are satisfied. (8 Marks)
3. Prove that a metric d induced by anorm on a normed space x satisfies;
4. d ( x +a, y + a) = d(x, y) (4 Marks)
5. d(dx, dy) = $\left|∝\right|$ d (x, y) for all x, y $\in x$ and every schalar $∝. $ (4 Marks)

**QUESTION FIVE (20 MARKS)**

a) (i) Define contraction in a metric space x. (2 Marks)

 (ii) State and prove the Banach fixed point theorem (contraction theorem) (10 Marks)

b) Prove that a mapping T of a metric space x into a metric space y is continuous if and only if the inverse image of any open subset of Y is an open subset of X (8 Marks)