MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

MAIN CAMPUS

CIT 106: LINEAR ALGEBRA

Date: 14th June, 2017 Time: 12.00 - 3.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Write your registration number on all sheets of the answer book used.
- Use a NEW PAGE FOR EVERY QUESTION attempted, and indicate number on the space provided on the page of the answer sheet.
- Fasten together all loose answer sheets used.
- Mobile phones and PDAs are NOT allowed in the examination room.

SECTION A: COMPULSORY QUESTION [30 MARKS]

QUESTION 1

a) Find a solution to the system (5 marks)

$$-x+y+z=-1$$
$$x+y+z=1$$
$$x+2y+z=2$$

b) Find the inverse of the matrix below (5 marks)

$$A=\begin{bmatrix}7 & -1 & 0\\ 0 & 0 & 4\\ 1 & 2 & -1\end{bmatrix}$$

c) Using elementary matrix multiplication, implement $5R_2 \rightarrow R_2$ for (5 marks)

$$A=\begin{bmatrix}2 & 1 & 1\\ 5 & 6 & -3\\ 4 & -1 & 1\end{bmatrix}$$

d) Find the solution of the system using Cramer's rule (5 marks)

$$7x-2y+z=15$$
$$x+y-3z=4$$
$$2x-y+5z=2$$

e) Find the eigenvectors of (5 marks)

$$z=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

are the eigenvectors of z a basis for C^2

f) Is the set $v_1=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $v_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$ complete? (5 marks)

SECTION B: ***ANSWER ANY TWO QUESTIONS*** ***[20 MARKS EACH].***

QUESTION 2

a) What is the elementary matrix that represents

$(i)\ 2R_2 + 7R_3 \to R_3$ for the matrix? (5 marks)

$$A = \begin{bmatrix} -1 & 0 & 4 \\ 5 & 2 & 0 \\ 8 & -7 & 1 \end{bmatrix}$$

$(ii)\ 5R_1 + 3R_2 \to R_2$ for the $2x2$ (5 marks)

$A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$ and then calculate the product EA

b) Write the vector (5 marks)

$u = (2i, 1+i, 3)$

as a linear combination of the set

$v_1 = (1,1,1)$, $v_2 = (1,0,-1)$, $v_3 = (1,-1,1)$

c) Write the polynomial (5 marks)

$v = 5t^2 - 4t + 1$

as a linear combination of the polynomials

$p_1 = 2t^2 + 9t - 1$, $p_2 = 4t + 2$, $p_3 = t^2 + 3t + 6$

QUESTION 3

a) Use row operations to put the matrix below into echelon form and find the rank. (5 marks)

$$B = \begin{bmatrix} 3 & 2 & -1 & 7 \\ 4 & 0 & 1 & 2 \\ 8 & 7 & -2 & 1 \end{bmatrix}$$

b) Find a parametric solution for the system. (5 marks)

$$5w - 2x + y - z = 0$$
$$2w + x + y + z = -1$$
$$-w + 3x - y + 2z = 3$$

c) Find the eigenvalues of (5 marks)

$$B=\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 0\\ 2 & 5 & 2\end{bmatrix}$$

d) Is the set $(-2,1,1)$, $(4,0,0)$, $(0,2,0)$ linearly independent? (5 marks)

QUESTION 4

a) Use Gauss-Jordan elimination to find the row canonical form of (5 marks)

$$A=\begin{bmatrix}2 & 2 & -1 & 6 & 4\\ 4 & 4 & 1 & 10 & 13\\ 8 & 8 & -1 & 26 & 23\end{bmatrix}$$

b) Find the commutator of the matrices (5 marks)

$$A=\begin{bmatrix}2 & 2 & -1\\ 4 & 0 & -1\\ 3 & 1 & 5\end{bmatrix} \quad , \quad B=\begin{bmatrix}1 & 3 & 1\\ 5 & 1 & 0\\ 3 & 0 & 0\end{bmatrix}$$

c) Find the trace of the matrix. (5 marks)

$$A=\begin{bmatrix}8 & 0 & 0 & -1\\ 7 & 9 & 1 & 0\\ 2 & 0 & 0 & 1\\ 9 & -8 & 17 & -1\end{bmatrix}$$

d) For the matrices

$$A=\begin{bmatrix}1 & -1 & 5\\ 0 & 4 & 0\\ 1 & 1 & -2\end{bmatrix} \quad , \quad B=\begin{bmatrix}9 & -1 & 0\\ 8 & 8 & 4\\ 16 & 0 & 1\end{bmatrix}$$

a. Find A^T and B^T (2 marks)

b. Show that $(A+B)^T=(A^T+B^T)$ (3 marks)

QUESTION 5

a) Solve the following system (4 marks)

$$3x - 2y + z = 9$$
$$4x + y + 3z = -1$$
$$-x + 5y + 2z = 7$$

b) Let $A = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 6 & 2 \\ 1 & 8 & 4 \end{bmatrix}$

a. Find the cofactors for this matrix (4 marks)
b. Find the adjugate of the matrix A (2 marks)
c. Calculate the determinant of matrix A (2 marks)
d. Find the inverse of matrix A (2 marks)

c) Let $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $v = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$, $w = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Find

i. The norm of each vector (3 marks)
ii. Normalize each vector (3 marks)

End of Exam