C. 1

MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

CITY CAMPUS

CIT 101: DISCRETE STRUCTURES I

Date: 14th June, 2017 Time: 5.30 - 8.30 pm

INSTRUCTIONS:

- Answer ALL question in SECTION A and any other TWO from SECTION B
- Write your registration number on all sheets of the answer book used.
- Use a NEW PAGE FOR EVERY QUESTION attempted, and indicate number on the space provided on the page of the answer sheet.
- Fasten together all loose answer sheets used.
- No mobile phones in the examination room.

QUESTION 1

a) Let Q(x, y) denote the statement "x = y + 3." What are the truth values of the propositions **Q(1, 2) and Q(3, 0)?** **(2 Marks)**

b) Show using truth tables that the following two Boolean formulae are logically equivalent. **(4 Marks)**

(i). $(A \cup \bar{B} \cup \bar{C}) \cap (A \cup B \cup \bar{C})$

(ii). $(A \cup \bar{C})$

c) Use contraposition to Prove that if n is an integer and $3n + 2$ is odd, then n is odd. **(5 Marks)**

d) Convert the decimal integer 11111010000_2 to decimal. Show your work.**(2 Marks)**

e) Consider these statements, of which the first three are premises and the fourth is a valid conclusion.

"All hummingbirds are richly colored."

"No large birds live on honey."

"Birds that do not live on honey are dull in color."

"Hummingbirds are small."

Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a hummingbird," "x is large," "x lives on honey," and "x is richly colored," respectively. Assuming that the domain consists of all birds, express the statements in the argument using quantifiers and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.f) **(6 marks)**

f) What is the Cartesian product $A \times B \times C$, where $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{0, 1, 2\}$? **(5 Marks)**

g) State the pigeon hole principal. **(2 Marks)**

h) Use set builder notation to give a description of each of these sets. **(4 marks)**

i) $\{0, 3, 6, 9, 12\}$

ii) $\{-3, -2, -1, 0, 1, 2, 3\}$

SECTION B: ***ANSWER ANY TWO QUESTIONS*** ***{20 MARKS EACH}.***

QUESTION 2:

a) Show that $\neg \forall x(P(x) \rightarrow Q(x))$ and $\exists x(P(x) \wedge \neg Q(x))$ are logically equivalent.

(5 Marks)

b) A group of 120 Kenyans numbered 1 to 120 had all even numbered Kenyans vying for Governorship, those whose numbers are divisible by 5 vying for Senator ship and those whose numbers are divisible by 7 vying for County representatives. How many vied for none of the three posts? **(5 Marks)**

c) Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \bar{A} \cup \bar{B}$ **(6 Marks)**

d) Let f_1 *and* f_2 be functions from **R** to **R** such that

$$f_1(x) = x^2 \text{ and } f_2(x) = x - x^2$$

What are the functions:-

(i) $f_1 + f_2$ **(2 marks)**

(ii) $f_1 f_2$ **(2 marks)**

QUESTION 3:

a) Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.

i) What is the composition of f and g? **(3 marks)**

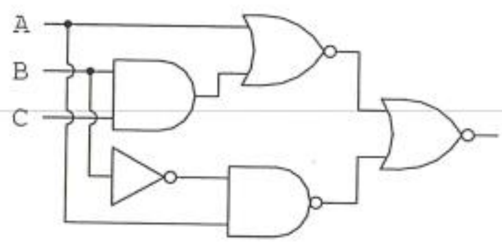
ii) What is the composition of g and f? **(3 marks)**

b) Determine whether each of these functions is a bijection from **R** to **R**.

i) $f(x) = -3x + 4$ **(2 marks)**

ii) $f(x) = -3x2 + 7$ **(2 marks)**

c) i) Give the outputs of the combined gates below. **(5 Marks)**



ii) Use Demorgan's theorems to simplify the output obtained in c) (i) above. Draw the gates for this final expression. **(5 Marks)**

QUESTION 4:

a) Show that if n is a positive integer, then $1 + 2 + 3 + \cdots . + n = \frac{n(n+1)}{2}$

(10 Marks)

b) A person takes at least one aspirin a day for 30 days. Using the idea of pigeonhole principal, show that if he takes 45 aspirin altogether, in some sequence of consecutive days he takes exactly 14 aspirin. **(4 Marks)**

c) A computer company receives 350 applications from computer graduates for a job planning a line of new Web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. Use the inclusion –exclusion principle to determine how many of these applicants majored neither in computer science nor in business **(6 Marks)**

OUESTION 5:

a) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit.

How many possible passwords are there? **(6 Marks)**

b) There are 18 mathematics majors and 325 computer science majors at a college.

(i) In how many ways can two representatives be picked so that one is mathematics major and the other is a computer science major?

(3 marks)

(ii) In how many ways can one representative be picked who is either mathematics major or a computer science major?

(3 marks)

c) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

(i) Exactly one vowel? **(2 marks)**

(ii) Exactly two vowels? **(2 marks)**

(iii)At least one vowel? **(2 marks)**

(iv)At least two vowels? **(2 marks)**

End of Exam