MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

CITY CAMPUS

CIT 106: LINEAR ALGEBRA

Date: 20th June, 2017 Time: 5.30 - 8.30 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.

CIT 106: LINEAR ALGEBRA TIME: 2 HOURS

Instruction: Answer Question One and any other two questions

Question 1 (30 Marks)

a) Find the determinant of the matrix

$$A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 8 & -9 \\ -1 & 7 & 0 \end{bmatrix}$$

(5 Marks)

b) Suppose that A is a 4×2 matrix, B is a 2×2 matrix, and X is a matrix such that $A = XB$.

i) What is the size of X? (2 Marks)

ii) Find a matrix X such that

$$\begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 3 \\ 4 & 1 \end{bmatrix} = X \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$$

(8 Marks)

c) Solve the following system of equations

$$x - y + z + w = 5$$

$$y - z + 2w = 8$$

$$2x - y - 3z + 4w = 18$$

(5 Marks)

d) Consider the following basis for $\mathbb{R}^2$:

$$E = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$

i) Find the coordinates for the vector $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ in terms of the basis E

(5 Marks)

ii) Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the following linear transformation:

$$L(x, y) = (2x - y, 3x - 2y)$$

Find the matrix representing L with respect to the basis E (5 Marks)

Question 2 (20 Marks)

a) Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{pmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

(10 Marks)

b) Consider the following matrix

$$A = \begin{bmatrix} 6 & 4 \\ -6 & -4 \end{bmatrix}$$

Find a general formula for the entries of A^n. (**Hint**: *Diagonalize* A)

(10 Marks)

Question 3 (20 Marks)

a) Determine the values of h and k for which the system

$$2x + hy = 8$$

$$x + 3y = k$$

i) has infinitely many solutions (5 Marks)

ii) no solution at all (5 Marks)

b) Describe the transformation defined by the matrix (5 marks)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

c) Determine the image of the point $(7, 2)$ and $(-3, -5)$under the said transformation (5 Marks)

Question 4 (20 Marks)

a) Suppose that $\begin{bmatrix}1\\1\end{bmatrix}$ is an eigenvector of a matrix A corresponding to the eigenvalue 3 and that $\begin{bmatrix}2\\1\end{bmatrix}$ is an eigenvector of A corresponding to the eigenvalue -2. Compute $A^2\begin{bmatrix}4\\3\end{bmatrix}$ (10 points)

b) Determine whether the vectors

$$\boldsymbol{v}_1 = \begin{bmatrix}1\\2\\3\\1\end{bmatrix};\ \boldsymbol{v}_2 = \begin{bmatrix}2\\2\\1\\3\end{bmatrix};\ \boldsymbol{v}_3 = \begin{bmatrix}-1\\2\\7\\-3\end{bmatrix}$$

are linearly independent or dependent. If the vectors are linearly dependent, find a non-trivial linear combination of them that is equal to the zero vector. (10 points)

Question 5 (20 Marks)

a) Determine whether **b** is a linear combination of vectors $\mathbf{a_1}, \mathbf{a_2}$and $\mathbf{a_3}$ where

$$\mathbf{a_1} = \begin{pmatrix}2\\-3\\4\end{pmatrix}, \mathbf{a}_2 = \begin{pmatrix}1\\6\\-1\end{pmatrix}, \mathbf{a}_3 = \begin{pmatrix}-1\\-1\\2\end{pmatrix}, \mathbf{b} = \begin{pmatrix}3\\-17\\17\end{pmatrix}$$

(7 Marks)

b) Write the augmented matrix corresponding to the system

$$x - 6y - 4z = -5$$
$$2x - 10y - 9z = -4$$
$$-x + 6y + 5z = 3$$

(3 Marks)

c) Solve the system in (b) above by applying the row reduction. If the system is consistent, find the general solution set. (10 Marks)