MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE CERTIFICATE IN BRIDGING MATHEMATICS

(CITY CAMPUS-REGULAR)

MMA 002: VECTORS AND GEOMETRY

Date: 1st December, 2015 Time: 2.00 - 4.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.

QUESTION ONE (Compulsory)

[30 Marks]

(a) Differentiate between:

(i) vector and scalar.

(ii) norm and unit vector. [2 Marks]

(b) Evaluate the following: [2 Marks]

(i) $(-2,3,5)-3(-7,-4,8)$

(ii) $-2(5,-8,4)+4(-4,0,-2)$ [2 Marks]

(c) (i) Differentiate between additive identity and additive inverse of a vector. [2 Marks]

(ii) Find the additive identity and additive inverse of the vector $\underset{\sim}{i}+3\underset{\sim}{i}-4\underset{\sim}{k}$. [2 Marks]

(d) (i) Differentiate between linear independence and linear dependence of vectors. [2 Marks]

(ii) Consider the vectors $\underset{\sim}{u}=(1,2,-1)$ and $\underset{\sim}{v}=(6,4,2)$ in $\mathbb{R}^3$. Show that $\underset{\sim}{w}=(9,2,7)$ is a linear combination of $\underset{\sim}{u}$ and $\underset{\sim}{v}$. [3 Marks]

(e) Given that vectors $\underset{\sim}{u}=3\underset{\sim}{i}-4\underset{\sim}{j}-2\underset{\sim}{k}$ and $\underset{\sim}{v}=\underset{\sim}{i}-4\underset{\sim}{j}+\underset{\sim}{k}$, find:

(i) $\underset{\sim}{u}\bullet\underset{\sim}{v}$

(ii) $\underset{\sim}{u}\times\underset{\sim}{v}$ [3 Marks]

[4 Marks]

(f) Normalize the following vectors:

(i) $\underset{\sim}{i}-3\underset{\sim}{j}+\underset{\sim}{k}$

(ii) $3\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k}$ [3 Marks]

[3 Marks]

QUESTION TWO

[20 Marks]

(a) Consider the vectors $\underset{\sim}{u}=(1,2,-1)$ and $\underset{\sim}{v}=(6,4,2)$ in $\mathbb{R}^3$. Show that $\underset{\sim}{w}=(0,8,-8)$ is a linear combination of $\underset{\sim}{u}$ and $\underset{\sim}{v}$, and that $\underset{\sim}{w}'=(4,-1,8)$ is not a linear combination of $\underset{\sim}{u}$ and $\underset{\sim}{v}$. [6 Marks]

(b) Use dot product to find the sides of the parallelogram with co-ordinates $(1,2)$, $(7,2)$, $(4,4)$ and $(10,4)$. [8 Marks]

(c) Determine whether the vectors $\underset{\sim}{i}=(1,0,0)$, $\underset{\sim}{j}=(0,1,0)$ and $\underset{\sim}{k}=(0,0,1)$ are linearly dependent or linearly independent. [6 Marks]

QUESTION THREE [20 Marks]

Let $\underline{u} = (3,2,-1)$, $\underline{v} = (0,2,-3)$ and $\underline{w} = (2,6,7)$. Compute

(a) $\underline{v} \bullet (4\underline{u} - \underline{w})$ [4 Marks]

(b) $\underline{u} \times (\underline{v} - 2\underline{w})$ [6 Marks]

(c) $(\underline{u} \times \underline{v}) \times (\underline{v} \times \underline{w})$ [10 Marks]

QUESTION FOUR [20 Marks]

(a) A man wishes to empty a cylindrical water tank with a diameter of 10 m. The outlet valve is on the opposite side of the tank, due north of him. Obtain:

(i) the displacement of the valve from his present location, and

(ii) the minimum distance he must walk in order to open the valve.

[3 Marks]

(b) Let $\underline{u} = (2,-2,3)$, $\underline{v} = (1,-3,4)$ and $\underline{w} = (3,6,-4)$. Evaluate the expression:

(i) $|\underline{u} + \underline{u} + \underline{v}|$ [3 Marks]

(i) $|-2\underline{u}| + 2|\underline{u}|$ [3 Marks]

(ii) $|3\underline{u} - 5\underline{v} + \underline{w}|$ [3 Marks]

(iii) $\left|\frac{1}{|\underline{w}|}\underline{w}\right|$ [3 Marks]

(c) Use diagrams to show the following:

(i) $\underline{u} + \underline{v} = \underline{v} + \underline{u}$ [2 Marks]

(ii) $\underline{u} + (\underline{v} + \underline{w}) = (\underline{u} + \underline{v}) + \underline{w}$ [3 Marks]

QUESTION FIVE [20 Marks]

(a) Differentiate between:

(i) commutative and associative property of vectors. [2 Marks]

(ii) position vector and displacement vector. [2 Marks]

(b) Find the distance between P_1 and P_2.

(i) $P_1(-3,6)$ and $P_2 = (-1,-4)$. [2 Marks]

(ii) $P_1(7,-5,1)$ and $P_2 = (-7,-2,-1)$. [3 Marks]

(c) Let $\underline{u} = (-1,2,5)$. Find all scalars k such that $|k\underline{u}| = 4$. [6 Marks]

(d) For which values of k do the following vectors become linearly dependent.

$\underline{u} = \left(k, -\frac{1}{2}, -\frac{1}{2}\right)$, $\underline{v} = \left(-\frac{1}{2}, k, -\frac{1}{2}\right)$, $\underline{w} = \left(-\frac{1}{2}, -\frac{1}{2}, k\right)$ [5 Marks]