MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF ARTS AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 100: BASIC MATHEMATICS

Date: 14th January, 2016 Time: 11.00 - 1.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Electronic scientific calculators may be used.
- Observe further instructions from the booklet.
- Cheating is NOT allowed and will be harshly punished.

QUESTION ONE (Compulsory) [30 Marks]

(a) (i) Define a null set and give an example [2 Marks]

(ii) Consider the following sets: $A = \{a, b, c, d\}$, $B = \{a, c\}$, $C = \{c, g, m, n, p\}$. Find

- the universal set U
- $(A \cap B^c) \cup C$

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- $A \cap A^c$. [3 Marks]

(b) How many 4-lettered word can be formed from the word the letters KHAQMEBD if the first and the third letters are to be vowels. Repeats not allowed. [2 Marks]

(c) If the sum of the first n terms of a certain series is $3^n - 1$. Find the first 4 terms of the series and state whether it is a G.P. or A.P. [3 Marks]

(d) A prize fund is set up with single investment of \$2000 to provide an annual prize of \$150. The fund accrues interest at 5% p.a. paid yearly. If the prize is awarded one year after the investment, find the number of years for which the full prize can be awarded. [5 Marks]

(e) Without using a log table or a calculator, evaluate $\sqrt{23}$ and round off your answer to four significant figures. [3 Marks]

(f) (i) Evaluate i^{49821} [2 Marks]

(ii) Express the complex number $-1 - i$ in polar form, giving any angles in terms of π radians. [3 Marks]

(g) If $\tan A = -\frac{1}{7}$ and $\cot B = \frac{4}{3}$, where A is obtuse and B is acute. Find $\cos(A + B)$, giving your answer in the form $\frac{p\sqrt{a}}{q}$. [4 Marks]

(h) Simplify $\log_5 t + \log_{\frac{1}{5}} t$. [3 Marks]

QUESTION TWO [20 Marks]

(a) Using set notations, prove that if A and B are subsets of some universal set, then

$$(A - B) \cup (B - A) = A \cup B - A \cap B.$$

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(b) Each day of a 5-day week, a boy takes an apple or an orange or a banana. [6 Marks]

(i) In how many different ways can he organize a week's fruit eating?

(ii) What would be the answer if he were not allowed to take the same fruit everyday?

(iii) What is the answer if he may not take the same fruits on any two consecutive days?

(c) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 3 & -1 & 3 \end{pmatrix}$ and hence solve for x, y, z in

$$\begin{aligned} x + y + 2z &= 3 \\ -x + 2y + z &= 9 \\ 3x - y + 3z &= 0. \end{aligned}$$

[8 Marks]

QUESTION THREE [20 Marks]

(a) Let S_n be the sum of the first n terms of an A.P. series.

(i) Show that S_n is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term and d is the common difference. [5 Marks]

(ii) Find the sum of the first 10 terms of $\sum_{r=1}^{n} \ln 2^r$ and the least value of n for which the sum of the first $2n$ terms exceeds 1000. [7 Marks]

(b) Expand $\frac{10}{(1+3x)(1-2x)}$ as a series of ascending powers of x giving the first 4 terms and the range for which the expansion is valid. [8 Marks]

QUESTION FOUR [20 Marks]

(a) Using Demoivre's theorem, find $(\sqrt{3}+i)^{13}$ and write your answer in the form $a+bi$. [4 Marks]

(b) Find the six distinct roots of the complex number $1+i\sqrt{3}$ and represent them graphically. [7 Marks]

(c) (i) State the remainder theorem. [2 Marks]

(ii) Let the polynomial $P(x)$ be given by $P(x) = x^3 + ax^2 + bx + c$. When $P(x)$ is divided by $x-1$, the remainder -8. If $x^2 - x - 2$ is a factor, find the values of a, b, c; hence solve $P(x) = 0$. [7 Marks]

QUESTION FIVE [20 Marks]

(a) Show that $\sin 3A = 3\sin A - 4\sin^3 A$. [4 Marks]

(b) Express $3\cos\theta + 4\sin\theta$ in the form $r\cos(\theta - \alpha)$; hence solve the equation $3\cos\theta + 4\sin\theta$. [8 Marks]

(c) Apply Cramer's rule to solve the following system of linear equations: [8 Marks]

$$\begin{aligned} -c + a + 3b &= 4 \\ 3a + 4c - 2b &= 11 \\ b + 2a + 3c &= 13. \end{aligned}$$

END

ALL THE BEST