MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF ARTS AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

CITY CAMPUS-SCHOOL BASED

MMA 102: CALCULUS I

Date: 10th February, 2016 Time: 2.00 - 4.00 pm

INSTRUCTIONS:

- Answer question ONE and any other TWO questions.
- Electronic scientific calculators may be used.
- Observe further instructions on the answer booklet.

QUESTION ONE (Compulsory) [30 Marks]

(a) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $g : \mathbb{R} \longrightarrow \mathbb{R}$ be given by $f(x) = \log x$, and $g(x) = \frac{1}{1-x^2}$.

(i) What are the domains of the functions f and g. [2 mks]

(ii) Determine the composite function $(f \circ g)(x)$, hence or otherwise, evaluate $(f \circ g)(-1)$. [2 mks]

(b) Evaluate the following limits;

(i) $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$. [2 mks]

(ii) $\lim_{n \to +\infty} \frac{x}{\sqrt{x^2+1}}$ [2 mks]

(iii) $\lim_{w \to -2} \frac{4-w^2}{w+2}$ [1 mk]

(c) Using the definition of continuity, discuss the continuity of the function

$$f(x) = \begin{cases} -x, & x < 0; \\ 2-x, & x \geq 0, \end{cases}$$

at the point $x = 0$. [2 mks]

(d) Find the derivative of the following functions from first principles:

(i) $f(x) = x^2$ [2 mks]

(ii) $f(x) = \sin x$ [2 mks]

(e) Determine the slope of the tangent to the curve $f(x) = \sqrt[3]{x}$ at the point $x = 8$; hence write the equation of this tangent line. [3 mks]

(f) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function with $f(0) = 5$ and $-1 \leq f'(x) \leq 3$ for all x. Show that $-5 \leq f(10) \leq 35$. [3 mks]

(g) (i) If a point traces the circle $x^2 + y^2 = 25$ and if $\frac{dx}{dt} = 4$ when the point reaches $(3, 4)$, find $\frac{dy}{dt}$ at this point. [2 mks]

(ii) A body moves in a straight line according to the law of motion $S = t^3 - 4t^2 - 3t$. Find its acceleration at each instant when the velocity is 0. [3 mks]

(h) Find $\frac{dy}{dx}$ if

(i) $y = \frac{1}{x}$ [1 mk]

(ii) $y = \frac{2}{x^4 \sqrt[4]{x}}$ [2 mks]

(iii) $y = \sec x$ [1 mk]

QUESTION TWO [20 Marks]

Find $\frac{dy}{dx}$ in the following functions

(a) $y^2 = \cos(\sin\sqrt{x^2+1})$ [4 mks]

(b) $x^2 = \frac{x-y}{x+y}$ [3 mks]

(c) $y = \cos^{-1} 2x$ [3 mks]

(d) $y = x^3 \ln 2x$ [3 mks]

(e) $y = x^{x^x}$ [4 mks]

(f) $y = \frac{(x^2+9)^2(x-3)^4}{x^2+2}$ [3 mks]

QUESTION THREE [20 Marks]

(a) By differentiating $x^2 - y^2 = 1$ implicitly, show that

$$y'' = -\frac{1}{y^3}$$

[5 mks]

(b) Write the polynomial

$$f(x) = -35 + 45x - 20x^2 + 3x^3$$

in the form

$$f(x) = C_0 + C_1(x-2) + C_2(x-2)^2 + C_3(x-2)^3$$

where the $C_i's$ $(i = 0, 1, 2, 3)$ are constants. [6 mks]

(c) Consider the curve $y = 4x^3 - 3x^4$

(i) Classify all the stationary points of the curve, and hence [5 mks]

(ii) sketch the curve [4 mks]

QUESTION FOUR [20 Marks

(a) Find the slope of the tangent to the curve $x^2 + xy + y^2 = 7$ at the point $(1, 2)$. Hence, fin