MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 103 LINEAR ALGEBRA I

Date: 21st April, 2016

Time: 8.30 - 10.30am

INSTRUCTIONS:

- **Answer Question ONE and any other TWO Questions.**

Question 1. **[Compulsory]** **(30 Marks)**

(a) Consider a non-empty subset $S = \{s_1, s, \ldots, s_n\}$ of a vector space V.

Define the following concepts;

(i) S is a subspace of V.

(ii) S is a linearly independent subset of V.

(iii) S is a basis for V **[6 Marks]**

(b) Solve the following system of linear equations using row reduction process.

$$x_1 + x_2 + 2x_3 = 8$$

$$-x_1 - 2x_2 + 3x_3 = 1$$

$$3x_1 - 7x_2 + 4x_3 = 10$$

[5 Marks]

(c) Let θ be the angle between the vectors **u** and **v** in the vector space $\mathbb{R}^2$.

Show that the scalar (dot) product of **u** and **v** is given by

$$\boldsymbol{u} \cdot \boldsymbol{v} = \|\boldsymbol{u}\|\|\boldsymbol{v}\| cos\theta \ .$$ **[5 Marks]**

(d) Given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 1 & 0 \end{bmatrix}$, find the

(i) reduced row echelon equivalence of A.

(ii) basis for the row space of A.

(iii) rank of A. **[6 Marks]**

(e) The points X =(-1 , 1, 2) , Y = (2 , -1, 4) and Z = (4, 3 , 1) lie on a

plane. Determine the

(i) symmetric equation of the line passing through X and Y.

(ii) equation of the plane containing the three points.

(ii) equation of the plane containing the three points.

(iii) area of the parallelogram whose consecutive vertices are X , Y and Z. [8 Marks]

QUESTION 2. (20Marks)

(a) Let vectors $\boldsymbol{u} = [x, y, z]$ in the subset V of $\mathbb{R}^3$ satisfy the following system of linear equations.

$$x + y + 2z = 0$$

$$x + 2y + z = 0$$

$$2x + 3y + 3z = 0$$

(i) Find a typical vector in V.

(ii) Show that V forms a vector subspace of $\mathbb{R}^3$ under the usual vector addition and scalar multiplication on the space $\mathbb{R}^3$.

(iii) Determine a basis and dimension of V. [12 Marks]

(b) Consider the system of linear equations;

$$x_1 + 2x_2 + x_3 = a$$

$$2x_1 + x_2 - x_3 = b$$

$$-4x_1 + x_2 + 5x_3 = c$$

(i) Given that a , b , c are non zero scalars , find the condition they must satisfy if the system is consistent.

(ii)Determine the basis and dimension of the solution space. [8 Marks]

Question3. **(20 Marks)**

(a) Let **u** and **v** be two vectors in $\mathbb{R}^3$. Show that $\|u+v\| \leq \|u\| + \|v\|$

[7 Marks]

(b) Given the vectors $u = i\text{-}2j\text{+}3k$; $v = \text{-}3i\text{+}2j\text{+}5k$, and $w = 2i\text{-}4j\text{+}k$; Find

(i) The projection of u in the direction of v.

(ii) The angle between u and w. **[6Marks]**

(c) Three consecutive vertices of a parallelogram are P =(1, 2 , 3),

Q = (-3, 1, 0) and R = (5 , 4 , -2). Find the

(i) equation of the plane containing the given parallelogram.

(ii) area of the parallelogram. **[7 Marks]**

Question 4. **(20 Marks)**

(a) (i) Let A be an $m \times n$ matrix and consider the mapping $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ defined by $T(x) = Ax$ for every n-vector x . Show that T is a linear transformation.

(ii) If the Kernel of T is the set $\{x \in \mathbb{R}^n : T(x) = o\}$. Show that the Kernel of T is a subspace of $\mathbb{R}^n$. **[10 Marks]**

b) A linear map T from the vector space $\mathbb{R}^3$ into itself is defined by

$T([x, y, z]) = [x + y, y + z, x - z]$

(i) Find the matrix A which represents T with respect to the standard

(b) A linear map T from the vector space $\mathbb{R}^3$ into itself is defined by

$T([x, y, z]) = [x + y, y + z, x - z]$

(i) Find the matrix A which represents T with respect to the standard ordered basis for $\mathbb{R}^3$.

(ii) Find the Kernel of T

(iii) Determine the vectors which span the range of T and the dimension of range of T. **[10 Marks]**

Question 5 **(20 Marks)**

(a) U and V are subspaces of a vector space W over a real field F.

(i) Show that intersection of U and V is also a vector subspace of W.

(ii) If U is $\{(x, y, z): x + y - z = 0\}$ and V is $\{(x, y, z): 2x + y + z = 0\}$.

Find a basis for the subspace formed by the intersection of U and V.

[12 Marks]

(b) Let W be the subspace of $\mathbb{R}^4$ spanned by the set
$V = \{[1,2,1,1], [0,1,-1,1], [1,0,2,3], [1,-1,2,6]\}$.

(i) Show that V is a linearly dependent set.

(ii) Obtain a subset of V which forms a basis for W.

(iii) Verify that the vector $v = [1, 5, -1, 0]$ is a vector in W.

[8 Marks]