UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

STA 223: OPERATION RESEARCH 1

DATE: NOVEMBER 29, 2016 **TIME: 11:00-1:00**

INSTRUCTIONS:

Answer Question ONE and ANY other TWO Questions

QUESTION ONE (30 MARKS)

a) Define the following terms in linear programming problem

i) Feasible solution (2 marks)

ii) Optimal feasible solution (2 marks)

iii) Regular basic solution (2 marks)

b) Suppose there are K types of food, the k^{th} type of food costs C_i per kilogram and X_i Kilogram are bought. Determine the function for cost of goods bought. (5 marks)

c) A farmer has 50 ha of land on which to plant maize and beans. He has a work force of 150 labourers and it takes 4 labourers to work on one ha of maize and 2 labourers to work one ha of beans. He has a capital of £4500 and 1 ha of maize requires £50 to cultivate and 1 ha of beans requires £100 to cultivate

Formulate linear programming problem by specifying the constraints. (4 marks) d). A linear program problem in standard form is given as: Optimize $Z=\underline{C}^T\underline{X}^T$

Subject to $A\underline{X} = \underline{b}$

With $\underline{X} \geq 0$

Given that that a linear program is given as: Maximize: $Z= 30x_1+ 40x_2$

Subject to: $x_1+ x_2 \leq 50$

$x_1+ 2x_2 \leq 80$

$2x_1+ x_2 \leq 90$

With $x_1, x_2 \geq 0$

i). Write this Linear program problem into standard form of linear program (2marks)

ii). State: $\underline{X}^T$ =(……..,…,) $\underline{C}^T$ =(……………) , $\underline{X}^T_B$ = (……………………)

$\underline{C}^T_B$ = (………………) and b = (…………………………) (2 marks)

iii). Determine matrix A (1 mark)

e). In a transportation model, decision variables are: x_{ij}= the size of the ship from warehouse i to out let j, where i= 1, 2, ……., m and j= 1, 2, ………n and for i and j the transportation cost per unit is given as c_{ij}

i). State the standard form of transportation problem for minimum cost and specify x_{ij} constraints.

(2 marks)

ii). suppose in a transportation problem of mxn, m=3 and n=2 and 5, a_1 =45, a_2=60, a_3=35, b_1=50, b_2=60, x_{11}=20, x_{12}=20, x_{13}=5, x_{21}=20, x_{22}=20 and x_{31}=10, x_{32}=20, x_{33}=5

Formulate this information in 3 x 3 tableau clearly showing the sinks and sources

(3 marks)

f). In a certain market only two types of food existed (A & B). One kilogram of A costs 4 Euros and contains 100 grams of protein and 5g of vitamins. A kilogram B costs 1 euro and contain 10g of protein and 5g of vitamins. A person has been recommended to consume 2kg of protein a week and 200g of vitamins. However his weekly food budget is only 150 euros.

Illustrate the protein vitamin and budget constrains graphically showing clearly the feasible set region (5 marks)

QUESTION TWO (20 MARKS)

a) Define the terms

(i) Convex set (1 marks)

(ii) Convex polytope (2 marks)

b) Show that a set of convex combination of points X_1, X_2 -------------- X_n € C where C is convex is also convex by adding two points Y_1 and Y_2 in C (5 marks)

c) (i) Write down the general formulation of the linear programming problems and write this in matrix form (3 marks)

(ii) A manufacturer produces two products X and Y. X has a contribution of $3 per unit and Y $4 per unit. The manufacturer wishes to establish the weekly production that maximizes contribution. The production data is as follows

Product	**Machining hrs/unit**	**Labour hrs/unit**	**Material kg/unit**
X	4	4	1
Y	2	6	1
Total available/week	100	180	4

Because of trade agreement, sales of X are limited to weekly maximum of 20 units and to honor an agreement with an old establishment customer at least 10 units of Y must be sold per week. Formulate as a linear programming problem and Determine the maximum contribution (9 marks)

QUESTION THREE (20 MARKS)

a) Define

(i) Simplex form of liner programming problem (1 mark)

(ii) Slack variable (2 marks)

b) Consider the following linear program (LP) problem and Minimize objective function P by

$P = -2x_1 + 3x_2$

Subject to

$4x_1 - x_2 \geq -10$

$2x_1 + 3x_2 \geq 12$

$X_1 \geq 0$ and x_2 is unconstrained

(i) Filling the table below with the final solution using simplex method (6 marks)

Basis	X1	X2	S1	S2	Solution	
S1						
X1						
X2						

(ii) Obtain the dual of the LP and state its optimal solution (2 marks)

(iii) Determine the range allowable change in the coefficient of x_2 that will not affect optimality of the given LP. (4 marks)

(iv) Suppose the resources of the first constraints increased by 5 and second one decreases by 8. Find the new optimal solution. (5 marks)

QUESTION FOUR (20 MARKS)

a) i. What are the characteristics of a transportation model. (3 marks)

ii. What is a northwest corner rule. (2 marks)

b) Suppose there are m resources and n tasks. Let X_{ij} be

$$X_{ij} \begin{cases} 1 \text{ if resource i is assigned task j} \\ 0 \text{ elsewhere} \end{cases}$$

State objective function for minimum cost subject to source and demand constraints.

(3 marks)

c) A company has four factory supply points (S_1, S_2, S_3 and S_4) and 3 warehouses located in the market area (D_1, D_2 and D_3). The capacity of supply points Ai are (350, 450, 520, 340) and demand of destinations (Bi) are (460, 450 and 480). The cost associated with transporting one unit of the resource from a supply point to a destination is given in the table below

	D_1	D_2	D_3
S_1	3	2	1
S_2	1	1	2
S_3	2	1	1
S_4	1	2	3

Determine

(i) Optimal allocation that minimize the transportation cost using Northwest corner rule (10 Marks)

(ii) Minimum transportation cost (2 marks)

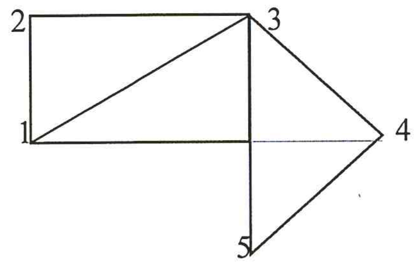
QUESTION FIVE (20 MARKS)

a) i What is graph theory (1 marks)

ii Distinguish between walk (w) and a trail (T) in a graph G (V, L) (2 marks)

iii Define adjacency matrix representation of a graph (3 marks)

b) Consider the graph below and



i Determine the adjacency matrix (3 marks)

ii What is the degree of vertexes 1 to 5 (2 marks)

d) Let A be the matrix adjacency of the graph

$$A= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(i) Determine A^2 (1 mark)

(ii) Give the number of walks of length 2 from:

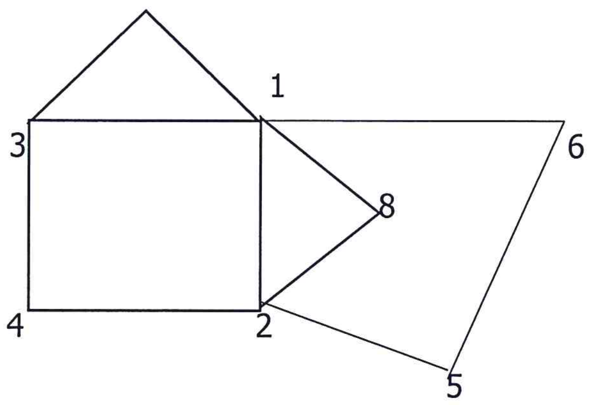
(1 to 1, 1 to 4, 4 to 1, 1 to 3) (2 marks)

e) i What is graph colouring (1 mark)

ii State for steps that are used to obtain independent subjects of a vertex set (2 marks)

iii Consider the graph below, colour it and determine the chromatic number of colours in.

(2 marks)



iv) Determine the number of colours (1 mark)

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