UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STA 222: TIME SERIES ANALYSIS 1

DATE: APRIL 5, 2018 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Define the following terms:

i) Time series (2 marks)

ii) Weakly stationary time series (2 marks)

b) Derive the normal equations of $P_t = xy^t$ using least square method (3 marks)

c) Fit the trend function to the following data on $Y_t = ab^t$ using least square method

t	1	2	3	4	5
X_t	1.6	4.5	13.8	40.2	135

Find the trend values (5 marks)

d) The first twenty sample autocorrelation coefficient of 400 random numbers are

$\hat{e}(1) = 0.07, \hat{e}(2) = 0.03, \hat{e}(3) = -0.17, \hat{e}(4) = 0.012, \hat{e}(5) = 0.033, \hat{e}(6) = 0.01, \hat{e}(7) = 0.23, \hat{e}(8) = 0.077, \hat{e}(9) = -0.07, \hat{e}(10) = -0.027, \hat{e}(11) = 0.04, \hat{e}(12) = -0.045, \hat{e}(13) = 0.0111, \hat{e}(14) = -0.117, \hat{e}(15) = 0.04, \hat{e}(16) = 0.043, \hat{e}(17) =$

$-0.08, \hat{e}(18) = 0.099, \hat{e}(19) = 0.57$ and $\hat{e}(20) = -0.09$. Is there evidence of non randomness. (2 marks)

e) Determine whether the process $X_t = e_t - 2e_{t-1} + e_{t-2}$ is invertible (4 marks)

f) Explain three properties of auto covariance and autocorrelation functions (3 marks)

g) Describe spectral analysis (3 marks)

h) Define Auto Regressive Moving Average Process of order (p,q) that is ARMA(p,q) in terms of α and β. Show that it can be expressed as $X_t = \frac{\theta(\beta)}{\theta(\alpha)}$ (3 marks)

i) Define Random Walk process and show that its Variance is time independent. (3 marks)

QUESTION TWO (20 MARKS)

a) Name and briefly describe the two mathematical model of time series analysis commonly used. (6 marks)

b) What is smoothing of the time series? And state two disadvantages of moving averages. (2 marks)

c) If $X_t = \frac{k}{1+e^{a+bt}}$; $k \neq 0$ and $b < 0$. Use a method of three ordinates (X_1, X_2 and X_3) at three equidistant values t_1, t_2 *and* t_3 to derive $\hat{k} = \frac{x_2^2(x_1+x_2)-2x_1x_2x_3}{x_2^2-x_1x_3}$ and $\hat{a} = \ln\left(\frac{\hat{k}}{x_1} - 1\right) - \hat{b}t_1$ (12 marks)

QUESTION THREE (20 MARKS)

a) Determine if the following processes are stationary or invertible

   i) $X_t = 2.5X_{t-1} + e_t$

   ii) $X_t = -0.8X_{t-1} + 0.5X_{t-2} + e_t$ (8 marks)

b) Consider an AR(2) process $X_t = \frac{3}{4}X_{t-1} - \frac{1}{8}X_{t-2} + e_t$. Is this process stationary? If so find its ACF (8 marks)

c) Define Moving average(X_t) of order q, that MA(q) and determine $E(X_t)$and $Var(X_t)$ (4 marks)

QUESTION FOUR (20 MARKS)

a) Explain the spectral density function $f(\lambda)$ of a given series X_t for which

$E(X_t) = 0$ $E(X_t X_{t+m}) = R(m)$, for all t. (4 marks)

b) Derive $f(\lambda) = \frac{1}{2\pi}\sum_{h=-\infty}^{\infty} R(h)\cos(\lambda h)$from $f(\lambda) = \frac{1}{2\pi}\sum_{h=-\infty}^{\infty} R(h)e^{-i\lambda h}$ (16 marks)

QUESTION FIVE (20 MARKS)

a) Define Auto Regressive process of order p[AR(q)] show that AR(1) is in fact an infinite order moving average process MA(∞) (10 marks)

b) Considering an AR(1), show that for $s > 0$, the best predictor of $\hat{X}_{n+s}$ is $\alpha^s X_n$

(10 marks)

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