UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR
FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STA 201: PROBABILITY AND STATISTICS I

DATE: DECEMBER 5, 2016 **TIME: 11:00AM-1:00PM**

INSTRUCTIONS:
Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Define the following terms

- i) Joint probability distribution (2 marks)
- ii) Bivariate probability distribution (2 marks)
- iii) Random variable (1 marks)

b) Show that the mean a geometric distribution is 1/p (5 marks)

c) Let S and T be distributed independently as exponential with means $1/\lambda$ and $1/\mu$ respectively. Let U = min{S, T} and V = max{S, T}. Find E(U) and E(U+V). (5 marks)

d) Let X be a random variable with U (0,1) distribution. Find the pdf of the random variable $Y = X / (1 + X)$ (5 marks)

e) State and Prove the Chebyshev's inequality theorem (5 marks)

f) Show that the mean of an exponential distribution is (a+b)/2 (5 marks)

QUESTION TWO (20 MARKS)

a) Let X be an exponential distribution with parameter λ then its density function is given by

$$f(X = x)\begin{cases}\frac{1}{\lambda}e^{-\frac{x}{\lambda}} & x > 0\\ 0 & elsewhere\end{cases}$$ (7 marks)

where $\lambda>0$

Find the

i) E(X)

ii) Var(X)

b) Let X be a Gamma distributed random variable with pdf given by

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\beta}}, x > 0, \ \beta > 0$$

Show that the

i) $E(X) = \alpha\beta$ (4 marks)

ii) $V(X) = \alpha\beta^2$ (5 marks)

c) Suppose that X is a Poisson random variable with mean λ. If $P(X = 2) = \frac{2}{3}P(X = 1)$, evaluate

i) P(X=0) (1 marks)

ii) P(X<3) (3 marks)

QUESTION THREE (20 MARKS)

a) Define the following terms

i) Moment generating function (2 marks)

ii) Statistical independence (2 marks)

b) Let X be a continuous random variable with probability distribution given by

$$f(x) = \begin{cases}\frac{3}{4}x^2(x-1) & 0 \le x \le 2\\ 0 & elsewhere\end{cases}$$

Calculate

i) The variance of $Y=4X^2+3X$ (8 marks)

c) A car hire firm has two cars which it hires out every day. The number of demands for a Car on each day is distributed as a Poisson random variable with mean of 1.5. Calculate the proportion of days on which

i) Neither car is used (3 marks)

ii) Some demand is refused (5 marks)

QUESTION FOUR (20 MARKS)

a) Two ideal dice are thrown. Let X_1 be the score on the first die and X_2 the score the second die. Let Y be the maximum score on die X_1 and X_2.

   i) Write down the joint distribution of Y and X_1 (4 marks)

   ii) Find the E(Y), Var(Y) and Cov(Y,X_2) (8 marks)

b) Prove that if X is normally distributed with mean μ and variance δ^2, then

$$M_X(t) = e^{\mu t + \frac{1}{2}\delta^2 t^2}$$

(3 marks)

c) State and prove the Markov inequality theorem (5 marks)

QUESTION FIVE (20 MARKS)

a) Let the joint pdf of X and Y be defined by

$$f(x,y) = \begin{cases} \frac{xy^2}{30} & X = 1,2,3; Y = 1,2 \\ 0 & Elesewhere \end{cases}$$

   i. Find the marginal pdf of X and Y (8 marks)

   ii. Show that X and Y are independent (3 marks)

b) Let Y be a random variable with mean 11 and a variance of 9. Using the Chebyshev's theorem, find the following

   i. A lower bound for $P(6<Y<16)$ (4 marks)

   ii. the value of C such that $P(|Y-11|\geq C)\leq 0.09$ (5 marks)

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