



# UNIVERSITY OF EMBU

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2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

STA 101 /SMA 140: INTRODUCTION TO PROBABILITY AND STATISTICS

**DATE: APRIL 5, 2016**

**TIME: 2:00-4:00 PM**

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**INSTRUCTIONS:**

**Answer Question ONE and ANY Other TWO Questions.**

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**QUESTION ONE (30 MARKS)**

- a) A binomial random variable has  $\mu=50$  and standard deviation  $=5$ , find the values of  $n$  and  $p$ . (4 marks)
- b) A coin is tossed three times in a row, what is the probability that all the tosses show the same side? (4 marks)
- c) Given that  $f(x) = \begin{cases} \frac{c}{\sqrt{x}}, & 0 < x < 4 \\ 0, & \text{elsewhere} \end{cases}$ , find the value of  $c$  (4 marks)
- d) A bag contains 3 blue balls, 4 red balls and 5 green balls. Find the probability that a ball picked at random from the bag is; (5 marks)
- i) green.
- ii) Not red.

e) Find the probability of getting 4 heads and 6 tails in 10 flips of a fair coin (4 marks)

f) The number of people who become ill each year from eating a kind of poisonous plant in a certain region is a random variable having a Poisson distribution with  $\lambda = 1.6$ , find the probability of; (5 marks)

- i. 2 such illnesses in a given year.
- ii. At least 7 such illnesses in 5 years

g) Let X have a p.d.f

$$f(x) = \begin{cases} x/6 & x = 1,2,3. \\ 0, & \text{elsewhere} \end{cases}$$

Find the mean and variance of  $x$  (4 marks)

### QUESTION TWO (20 MARKS)

a) Estimate the mean and standard deviation from the frequency distribution shown below, (10 marks)

Diameter(mm)	5-9	10-14	15-19	20-24	25-29	30-34
Frequency(f)	4	9	16	12	6	3

b) The probability that I hit the bull's eye in a dart game is 0.12.find the probability in 8 trials, I hit it: (10 marks)

- i. Exactly 4 times
- ii. At least once.
- iii. What is the  $E(x)$

**QUESTION THREE (20 MARKS)**

- a) Three different machines in a factory have different probabilities of breaking down during a shift as shown, (8 marks)

machine	Probability of breaking
A	$\frac{4}{15}$
B	$\frac{3}{10}$
C	$\frac{2}{11}$

Find,

- i. The probability that all machines will break down during one shift.
  - ii. The probability that none of the machines will break down during the shift
- b) If  $x$  is normally distributed with parameters  $\mu=10, \delta^2 = 36$  Compute; (6 marks)
- i)  $P(x > 5)$ ,
  - ii)  $P(4 < x < 6)$
- c) Given the frequency table below find:- (6 marks)
- i) First quartile
  - ii) Fifth decile
  - iii) 80<sup>th</sup> percentile

Diameter(mm)	10-15	15-20	20-25	25-30	30-35	35-40
Frequency(f)	20	15	31	22	10	2

**QUESTION FOUR (20 MARKS)**

- a) The daily water usage per person in Embu is normally distributed with a mean of 20 gallons and a standard deviation of 5 gallons. What is the probability that a person from Embu selected at random will use; (12 marks)
- Less than 20 gallons per day?
  - More than 30 gallons per day?
  - What percent of the population uses between 25 and 30 gallons?
- b) The probability that a car will break down while driving through a certain tunnel is 0.0004. Find the probability that among 2000 cars driving through the tunnel, at most one will break down. (8 marks)

**QUESTION FIVE (20 MARKS)**

- a) Given the joint pdf  $f(x, y) = \begin{cases} 2/(1+x+y)^3, & 0 < x < y < 1 \\ 0, & \text{elsewhere} \end{cases}, x > 0, y > 0$ , find the marginal densities of X and Y. (6 marks)
- b) Prove that the mean and variance of a Poisson distribution is  $\lambda$ . (8 marks)
- c) Suppose X and Y are continuous random variables with joint pdf

$$f(x, y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 < x < 2, \\ & 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the  $p(x < 1, y < 3)$  (6 marks)

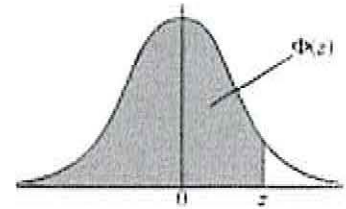
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## THE NORMAL DISTRIBUTION FUNCTION

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $z$ , the table gives the value of  $\Phi(z)$ , where

$$\Phi(z) = P(Z \leq z).$$

For negative values of  $z$  use  $\Phi(-z) = 1 - \Phi(z)$ .



z											ADD								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0



