UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 425: PARTIAL DIFFERENTIAL EQUATIONS I

DATE: APRIL 4, 2017 **TIME: 11:00AM-1:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Define the following terms: (3 marks)

i) Linear partial differential equation

ii) Semi-linear partial differential equation

iii) Quasi-linear differential equation

b) Determine the necessary and sufficient condition of compatibility of two non-linear first order partial differential equations (4 marks)

$$f(x,y,z,p,q)=0$$
$$g(x,y,z,p,q)=0$$

c) Form a partial differential equation by eliminating the arbitrary constants a and b in the given function. (5 marks)

$$2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$

d) Find the general solution of the partial differential equation (4 marks)

$$3p+5q=2$$

e) Solve the non-linear partial differential equation (5 marks)

$$pq+qx=y$$

f) Show that the direction cosines of the tangent at the point (x,y,z) to the conic (5 marks)

$ax^2+by^2+cz^2=1, x+y+z=1$ are proportional to $(by-cz, cz-ax, ax-by)$

g) If u is a function of x, y and z which satisfies the partial differential equation (4 marks)

$$(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$$

Show that u contains x, y and z only in the combinations $x+y+z$ and $x^2+y^2+z^2$

QUESTION TWO (20 MARKS)

a) Find complete and singular solutions to the given differential equation (7 marks)

$$z=px+qy+p^2+pq+q^2$$

b) Find the condition that the plane $lx+my+nz+p=0$ should touch the central concoid (8 marks)

$$ax^2+by^2+cz^2=1$$

c) Solve the given non-linear partial differential equation and identify singular point if any. (5 marks)

$$p^2+q^2=9$$

QUESTION THREE (20 MARKS)

a) Find a differential equation arising from

$$\phi(x+y+z, x^2+y^2-z^2)=0$$

(8 marks)

b) Find general solution of the partial differential equation

$$(y-z)p+(x-y)q=z-x$$

(7 marks)

c) Solve the non linear differential equation

$$p-q=x^2+y^2$$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Solve for the complete and singular solutions in the given differential equation

$$4(1+z^3)=9z^4pq$$ (8 marks)

b) By use of transformation method solve the differential equation

$$4xyz=pq+2px^2y+2qxy^2$$

(7 marks)

c) Find the integral curves of the equation (5 marks)

$$\frac{dx}{y-x}=\frac{dy}{x+y}=\frac{zdz}{x^2+y^2}$$

QUESTION FIVE (20 MARKS)

a) Find the general integral surface of the quasi-linear partial differential equation

$$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$$ (10 marks)

b) Derive the condition of integrability of the shown Pfaffian differential equation .

$$(y^2+yz)dx + (xz+z^2)dy + (y^2-xy)dz = 0$$

Hence, show that keeping z constant, the integration of the equation yields the following relation.

$$\frac{y(x+z)}{y+z} = F(z)$$ where, $F(z)$ is an arbitrary function of z (10 marks)

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