UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 408: COMPLEX ANALYSIS II

DATE: DECEMBER 7, 2016 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS) -COMPULSORY

a) Define the following terms
   i. Analytic function (2 marks)
   ii. Harmonic function (2 marks)

b) Given that $F(z) = z^2$. Show that $F(\bar{z}) = \overline{F(z)}$ (4 marks)

c) Define a conformal mapping. (2 marks)

d) Find the residues of $F(z)$ at each of its poles given that $F(z) = \frac{e^z}{(z-2)(z-4)}$ (4 marks)

e) State without proof the Argument principle. (4 marks)

f) Find the order of the pole of $\frac{z^2+1}{e^z+1}$ at $z = \pi i$ (4 marks)

g) Show that the function $f(z)=2x^2+y+i(y^2-x)$ is not analytic at any point (5 marks)

h) Discuss the singularity of $f(z) = \frac{e^z}{\sin z}$. (3 marks)

QUESTION TWO (20 MARKS)

a) Given the function $U(x, y) = x^3 - 3xy^2 - 5y$
   (i) show that u is harmonic in the entire complex plane (4 marks)
   (ii) Find the harmonic conjugate function u (4 marks)

b) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a Laurent series valid for the following annular domains
   (i) $0<|z-1|<2$ (4 marks)
   (ii) $0<|z-3|<2$ (5 marks)

c) State without proof the Cauchy residue theorem. (3 marks)

QUESTIONS THREE (20 MARKS)

a) Show that $F_1(z) = z - z^2 + z^3 - z^4 + z^5$ ...is analytic in |z|<1 and hence find a function that represents all analytic continuation of F(z) (7 marks)

b) Evaluate $\int_0^\infty \frac{2x^2-1}{x^4+5x^2+4}dx$ (7 marks)

c) Find the residue of f (z) $= \frac{\tan z}{z^2+z+1}$ at all singularities of tan z. (6 marks)

QUESTION FOUR (20 MARKS)

a) Evaluate $\int_o^{2\bar{x}} \frac{d\theta}{5+3\sin\theta}$ (8 marks)

b) By considering the lines $x = a$ and $y = b$ in the z-plane, show that $f(z) = e^z$ is conformal at every point in the whole complex plane. (9 marks)

c) Prove that for closed polynomials the sum of the exponents

$$\left(\frac{\propto_1}{\pi} - 1\right), \quad \left(\frac{\propto_2}{\pi} - 1\right) .... \left(\frac{\propto_n}{\pi} - 1\right)$$

In the Schwarz Christofell transformation is equal to -2. (3 marks)

QUESTIONS FIVE (20 MARKS)

a) i) Define what is meant by an infinite product P_n converges. (2 marks)

ii) Show that the infinite product $\prod_{k=1}^{\infty} \left(1 - \frac{z^3}{k^3}\right)$ converges. (5 marks)

b) Show that $\prod_{k=1}^{\infty} \left(1 + \frac{1}{\sqrt{k+1}}\right)$ diverges. (4 marks)

c) Evaluate $\int_0^\infty \frac{e^{2x}}{(x-2)^2\,(x^2+1)}dx$ (7 marks)

d) Show that $F(z) = z^2 + i$ does not satisfy the Shwarz reflection principle. (3 marks)

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