UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

SMA 405: ALGEBRA II

DATE: APRIL 3, 2018 **TIME: 8:30 AM – 10:30 AM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a)

i) Derive a group table/ Cayley table in $\mathbb{Z}_3$ (2 marks)

ii) Name three families of group and identify among them the finitely generated group among them. (4 marks

b) Show that an integral domain $\mathbb{Z}$ is not a field (4 marks)

c) Show that a group of order 200 has a normal Sylow 5 subgroup. (5 marks)

d) Define Euclidean in integral domain and state its properties (5 marks)

e) Define Field extension and list four examples of a ring (5 marks)

f) Show that $f(q) = q^4 + 2q^2 + 1$ has no root in $\mathbb{R}$ but has two distinct roots in X. (5 marks)

QUESTION TWO (20 MARKS)

a)

i) Define multiplicity an of roots in polynomial (2 marks)

ii) Find all of the zeros and multiplicity of

$\mathrm{f}(\mathrm{x}) = 4x^8 - 64x^7 + 445x^6 - 1757x^5 + 4310x^4 - 6728x^3 + 6528x^2 - 3600x + 864$,

where $x \varepsilon \mathbb{R}$ (8 marks)

b)

i) Define reducibility and irreducibility of a polynomial (2 marks)

ii) Determine whether the functions $\mathrm{f}(y) = y^6 - 27y^4 + 243y^2 + 729$ and $\mathrm{g}(x) = y^3 + 6$ are reducible or not. $x \varepsilon \mathbb{R}$ or $\mathbb{C}$ (8 marks)

QUESTION THREE (20 MARKS)

a)

i) Define a field (2 marks)

ii) Proof that If R is a field and $xy = 0$ in P for some x,y with $x \neq 0$; then R is an integral domain (3 marks)

iii) Let $z \varepsilon \mathbb{C}$ be defined as $z = a + bi$ where $\{a, b\} \varepsilon \mathbb{R}$ and $z \neq 0$. Use Z to check whether $\mathbb{C}$ is field or not? (5 marks)

b) State and Proof 1st Sylow Theorem (10 marks)

QUESTION FOUR (20 MARKS)

a)

i) Define a ring and state its axioms (3 marks)

ii) Let Region R be a set of $\mathbb{R}$ or $\mathbb{Z}$ or $\mathbb{C}$and let R(p) denotes polynomials in p whose coefficients in set of real numbers.
If $\mathrm{f}(\mathrm{p}) = 2\mathrm{p}^3 + \mathrm{p}^2 - 5$, $\mathrm{g}(\mathrm{p}) = \mathrm{p}^2 - 4\mathrm{p} + 7$ and $\mathrm{h}(\mathrm{p}) = 2\mathrm{p}$, Use f, g and h to verify axioms in a(i) (7 marks)

b) State and Proof 2nd Sylow Theorem (10 marks)

QUESTION FIVE (20 MARKS)

a) State and Proof 3rd Sylow Theorem (10 marks)

b)

i) Define internal and external direct product of groups (4 marks)

ii) Give two examples of internal and external direct product of groups (6 marks)

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