UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR
FIRST SEMESTER EXAMINATION

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FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE,
SMA 401: TOPOLOGY I

DATE: DECEMBER 5, 2016 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY other TWO Questions

QUESTION ONE (30 MARKS)

a) Define what is meant by a topology on a set X. (3 marks)

b) Is $\tau_1 = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}$ a topology over the set $X = \{1,2,3,4,5\}$. Justify your answer. (3 marks)

c) Let f: $X \to Y$ and g: $Y \to Z$ be continuous functions. Show that the composite function $(g \circ f)$ is also continuous. (4 marks)

d) Let (X,τ) be a topological space. Denote a derived set of A by $A^{|}$. Prove that a subset $A \subset X$ is closed if and only if $A^{|} \subset A$. (5 marks)

e) (i) Define a T_4- space. (1 mark)

(ii) Let X be a T_4- space. Then show that X is also a regular space. (3 marks)

f) Let $\tau = \{X, \emptyset, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology defined on $X = \{a,b,c,d,e\}$. Find the closure of $\{b,d\}$. (4 marks)

g) Let (X,τ) be a topological space. Prove that a subset $A \subset X$ is open if and only if its complement $X \setminus A$ is closed. (3 marks)

h) Let β_1 *and* β_2 be bases respectively for topologies τ_1 and τ_2. Suppose each open set B$\in \tau_1$ is the union of members of β_2, show that τ_1 is coarser thanτ_2 . (4 marks)

QUESTION TWO (20 MARKS)

a) Define a limit point of a subset A of a topological space X. (1 mark)

b) Let X = {a,b,c,d,f} and $\tau = \{X, \emptyset, \{a\}$, {b,c}, {a,b,c}, {c}, {a,f}, {f}, {b,c,f}, {a,b,c,f}}. Let A = {a,c,d}. Show that b is a limit point of A but a is not. (5 marks)

c) Show that if X is a discrete topological space and that A$\subset X$, then the derived set of A is empty. (4 marks)

d) Let X be a topological space and A, B $\subset X$. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$. (3 marks)

e) Let X be a topological space and A, B $\subset X$. Denote A° the interior of A.

i. Using an appropriate example show that $A^\circ \cup B^\circ \neq (A \cup B)^\circ$. (3 marks)

ii. Prove that $A^\circ \cap B^\circ = (A \cap B)^\circ$. (4 marks)

QUESTION THREE (20 MARKS)

a) Show that the the property of a regular space is hereditary. (5 marks)

b) Let p $\in X$and denote $N_p$the set of all neighborhood of a point p. Prove the followings

i. For all pairs N, M $\in N_p$, N$\cap$M $\in N_p$. (3 marks)

ii. If N $\in N_p$and for every M $\subset X$ with N $\subset M$ it implies that M $\in N_p$. (3 marks)

c) Consider the following topology on X = {a,b,c,d,e}; $\tau = \{X, \emptyset, \{a\}$, {a,b}, {a,c,d}, {a,b,c,d}, {a,b,e}}. If A = {a,b,c} find

i. The exterior of A. (3 marks)

ii. The boundary of A. (3 marks)

iii. Hence show that the boundary of A, $\delta(A) = \bar{A} \cap \overline{(X/A)}$. (3 marks)

QUESTION FOUR (20 MARKS)

a) Let F: X $\rightarrow Y$ be a bijective function. Prove the following statements are equivalent:

i.) F is a homomorphism.

ii.) F is open.

iii.) F is closed.

iv.) F$(\bar{A}) = \overline{F(A)}$. (10 marks)

b) Let P: X → Y be a closed map. Let S ⊂ Y be any subset of Y and U⊂ X be any open set in X such that P^{-1}(S) ⊂ U, prove that there exists an open set V ⊂ Y such that S ⊂ V and P^{-1}(S) ⊂ P^{-1}(V) ⊂ U. (4 marks)

c) Let θ be a subbasis for the topologies τ and τ^*, then show that $\tau = \tau^*$. (6 marks)

QUESTION FIVE (20 MARKS)

a) Show that every point p in a discrete topological space X has a finite local base. (4 marks)

b) Prove that a topological space X is a T_1 space if and only if every singleton subset $\{p\} \subset X$ is closed. (6 marks)

c) Let (X, τ_x) nd (Y, τ_y) be topological spaces, show that that F: X→ Y is closed if and only if $\overline{F(A)} \subset F(\overline{A})$ for all A⊂ X. (5 marks)

d) Let X = {a,b,c,d} with $\tau_x = \{X, \emptyset, \{a,b\}, \{a\}, \{b\}\}$ and let Y = {x,y,z,t}with τ_y = $\{Y, \emptyset, \{x\}, \{y\}, \{x,y\}\}$. Define the function F as F(a) = x, F(b) = y, F(c) = z and F(d) = t. Show that F is a homomorphism. (5 marks)

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