UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR
FIRST SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE/ARTS), BACHELOR OF SCIENCE (INDUSTRIAL CHEMISTRY) AND BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

SMA 371: LINEAR PROPGRAMMINGI

DATE: DECEMBER 1, 2016 **TIME: 8:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY other TWO Questions

QUESTION ONE (30 MARKS)

a) Define the following terms

(i) Linear programming (2 marks)

(ii) Duality (2 marks)

(iii) Transport problem (3 marks)

(iv)Unbounded solution (3 marks)

b) Solve graphically and determine the feasible region (5 marks)

Minimize $Z = 10x_1 + 4x_2$

Subject to

$3x_1 + 2x_2 \geq 60$

$7x_1 + 2x_2 \geq 84$

$3x_1 + 6x_2 \geq 72$

$x_1 \geq 0$, $x_2 \geq 0$

c) For the linear programming problem of

$$Min\ 5x_1 + 2x_2 + 3x_3 + x_4$$

subject to

$$x_1 + x_2 - 2x_3 + 3x_4 = 2$$

$$-2x_1 - x_3 = 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Show geometrically that there can be only two basic feasible solutions to the problem.

(4 marks)

d) Define the tern linear programming problem and giving any two areas of its applications

(5 marks)

e) Consider the following Linear Programming model

$$\text{Max } Z = x_1 + 5x_2 + 3x_3$$

$$x_1 + 2x_2 + x_3 = 3$$

$$2x_1 - x_2 = 4$$

$$x_1, x_2, x_3 >= 0$$

Given that basic variables are x_1 and x_3 in the primal optimal solution, find the dual optimal solution (dual variables value and dual objective function value) without solving the dual model. (5 mark)

f) State the dual for the following Linear programming problem (5 mark)

$$\text{Min } 17x_1 + 29x_2 + x_4$$

$$2x_1 + 3x_2 + 2x_3 + 3x_4 \leq 40$$

$$4x_1 + 4x_2 + x_4 \geq 10$$

$$3x_3 - x_4 = 0$$

$$x_1, x_2, x_3, x_4 \geq 0$$

QUESTION TWO (20 MARKS)

a) Define the following linear programming terminologies (3 marks)

i. Feasible Solution

ii. Basic Feasible Solution

iii. Optimum Solution

b) Consider the linear program

$\text{Min } Z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to

$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

i. State the above in canonical form (2 marks)

ii. Solve by the simplex method (7 marks)

c) Prove the theorem. If x is feasible for the standard maximum problem max c'x subject to the constraint $Ax \leq b$ and $x \geq 0$ and if y is feasible for its dual min y'b subject to constraint $y'A \geq c'$ and $y' \geq 0$, then $c'x \leq y'b$. (3 marks)

d) Consider the following Linear Programming model

$\text{Max } x_1 + 5x_2 + 3x_3$

$x_1 + 2x_2 + x_3 = 3$

$2x_1 - x_2 = 4$

$x_1, x_2, x_3 >= 0$

i. Find the dual of the model. (2 marks)

ii. Given that basic variables are x_1 and x_3 in the primal optimal solution, find the dual optimal solution (dual variables value and dual objective function value) without solving the dual model. (3 marks)

QUESTION THREE (20 MARKS)

a) State the fundamental theorem of linear programming (4 marks)

b) Consider the following product mix problem: Three machine shops A, B, C produces three types of products X, Y, Z respectively. Each product involves operation of each of the machine shops. The time required for each operation on various products is given as follows:

Products\Machine shops	A	B	C	Profit per unit
X	10	7	2	12
Y	2	3	4	3
Z	2	3	4	1
Available Hours	100	77	80	

The available hours at the machine shops A, B, C are 100, 77, and 80 only. The profit per unit of products X, Y, and Z is \$12, \$3, and \$1 respectively.

i Define a linear programming problem (3 marks)

ii Find the optimal solution of the problem (6 marks)

c) A company owns 2 oil mills A and B which have different production capacities for low, high and medium grade oil. The company enters into a contract to supply oil to a firm every week with 12, 8, 24 barrels of each grade respectively. It costs the company Ksh 1000 and Ksh 800 per day to run the mills A and B. On a day A produces 6, 2, 4 barrels of each grade and B produces 2, 2, 12 barrels of each grade. Formulate an LP problem to determine number of days per week each mill will be operated in order to meet the contract economically. (7 marks)

QUESTION FOUR (20 MARKS)

a) Define the term

i. Duality (2 marks)

ii. Slack variable (2 marks)

iii. Sensitivity analysis (3 marks)

b) State and prove the weak duality theorem (7 marks)

c) If the primal problem is

Maximize $Z=3x_1+2x_2+x_3$

Subject to $x_1+2x_2-x_3\leq 4$

$2x1-x2+x3=8$

$x_1-x_2\leq 6$

$x_1, x_2, x_3\geq 0$

Find its dual problem (3 marks)

d) Distinguish between mathematical programming and linear programming (3 marks)

QUESTION FIVE (20 MARKS)

A company manufactures and sells two types of product to a number of customers. The company is currently preparing its budget for the year ending 31 December 2010 which it divides into 12 equal periods. The cost and resource details for each of the company's product types are as follows:

	Product type M	Product type F
	$	$
Selling price per unit	200	210
Variable costs per unit		
Direct material P ($2.50 per litre)	20	25
Direct material Q ($4.00 per litre)	40	20
Direct labour ($7.00 per hour)	28	35
Overhead ($4.00 per hour)	16	20
Fixed production cost per unit	40	50
	Units	Units
Maximum sales demand in period 1	1,000	3,000

The fixed production cost per unit is based upon an absorption rate of $10 per direct labour hour and a total annual production activity of 180,000 direct labour hours. One-twelfth of the annual fixed production cost will be incurred in period 1. In addition to the above costs, non-production overhead costs are expected to be $57,750 in period 1. During period 1, the availability of material P is expected to be limited to 31,250 litres. Other materials and sufficient direct labour are expected to be available to meet demand.

It is the company's policy not to hold inventories of finished goods.

a) Calculate the number of units of product types M and F that should be produced and sold in period 1 in order to maximize profit. (3 marks)

b) Using your answer to (a) above, prepare a columnar budgeted profit statement for period 1 in a marginal cost format. (4 marks)

After presenting your statement to the budget management meeting, the production manager has advised you that in period 1 the other resources will also be limited. The maximum resources available will be:

Material P	31,250 litres
Material Q	20,000 litres
Direct labour	17,500 hours

It has been agreed that these factors should be incorporated into a revised plan and that the objective should be to make as much profit as possible from the available resources.

c) Using graphical linear programming to determine the revised production plan for period 1. State clearly the number of units of product types M and F that are to be produced.

(7 marks)

d) Using your answer to part (c) above, calculate the profit that will be earned from the revised plan. (2 marks)

e) Calculate and explaining the meaning of the shadow price for material Q. (4 marks)

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