UNIVERSITY OF EMBU

2016/2017 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE/ARTS), BACHELOR OF SCIENCE (INDUSTRIAL CHEMISTRY) AND BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

SMA 341: PROBABILITY AND STATISTICS II

DATE: NOVEMBER 30, 2016 **TIME: 2:00-4:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Define the terms

   i. indicator random variable (2 marks)

   ii. probability generating function (2 marks)

b) Suppose that X and Y are two random variables with finite second moments. Also, assume that

   $P(X + Y = 0) < 1$. Show that

   $$(E\{(X+Y)^2\})^{1/2} \leq (E(X^2))^{1/2} + (E(Y^2))^{1/2}.$$

   (4 marks)

c) Consider two random variables X_1, X_2 whose joint pdf is given by

   $f(x_1,x_2) = \exp\{-\theta x_1 - \theta^{-1}x_2\}I(x_1 > 0 \cap x_2 > 0)$

   with $\theta(>0)$. Are X_1, X_2 independent? (5 marks)

d) Show that $P(A) \in [0,1]$ (3 marks)

e) state and prove the central limit theorems (5 marks)

f) Let the moment generating function of a discrete random variable X be given by $M_x(t) = 0.25e^{t} + 0.35e^{3t} + 0.4e^{5t}$ Find P(X=3). (4 marks)

g) Let X be a geometric random variable with parameter p.

i) Determine the probability generating function of X. (3 marks)

ii) Find the mean of X for p=1/2. (2 marks)

QUESTION TWO (20 MARKS)

a) You are given the random vector $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}$ with mean vector $\mu = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 3 \\ 0 \end{bmatrix}$ and variance-covariance matrix $\Sigma = \begin{bmatrix} 4 & -1 & 0.5 & -0.5 & 0 \\ -1 & 3 & 1 & -1 & 0 \\ 0.5 & 1 & 6 & 1 & -1 \\ -0.5 & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$.

Partition X as $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \hline X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} X^{(1)} \\ \hline X^{(2)} \end{bmatrix}$. Let $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$

i) $E(BX^{(2)})$ (3 marks)

ii) $Cov(AX^{(1)})$ (3 marks)

iii) $Cov(X^{(1)}, X^{(2)})$ (3 marks)

iv) $Cov\left(AX^{(1)}, BX^{(2)}\right)$ (3 marks)

b) A car dealer sells X cars each day and always tries to sell an extended warranty on each of these cars. (In our opinion, most of these warranties are not good deals.) Let Y be the number of extended warranties sold; then $Y \leq X$. The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} c(x+1)(4-x)(y+1)(3-y) & x = 0,1,2,3; y = 0,1,2 \text{ with } y \leq x \\ 0 & \text{elsewhere} \end{cases}$$

i) Find the value of c. (3 marks)

ii) find the marginal pdf of X and Y (4 marks)

iii) Are the X and Y independent (1 mark)

QUESTION THREE (20 MARKS)

a) The joint pdf of a bivariate random variable (X,Y) is given by

$$f(x,y) = \begin{cases} \frac{1}{y} e^{-x/y} e^{-y}, & x > 0, \quad y > 0 \\ 0, & otherwise \end{cases}$$

i) Show that f(x, y) is a probability density function. (4 marks)

ii) Find P(X>1|Y=y). (5 marks)

b) Let the moment of a discrete random variable X be given by

$$E(X^k) = 0.8, \qquad k = 1,2, \dots \dots \dots$$

Find the moment generating function of X. (4 marks)

c) Find the variance covariance matrix for the two random variables X_1 and X_2 when the joint probability distribution $P(X_1, X_2)$ is represented by the entries in the table below

(7 marks)

X_2 \ X_1	0	1	$P(X_1)$
-1	0.4	0.06	0.3
0	0.16	0.14	0.3
1	0.40	0.00	0.4
$P(X_2)$	0.8	0.20	1

QUESTION FOUR (20 MARKS)

a) If the independent random variables X and Y are distributed as N(0,1), set U=X+Y, V=X-Y.

   i) Determine the pdfs of U and V. (6 marks)

   ii) Show that U and V are independent (3 marks)

   iii) Calculate the probability that $U<0$ and $V<0$ (4 marks)

b) State and prove the Bonferroni's inequality (7 marks)

QUESTION FIVE (20 MARKS)

a) Let X_1, X_2, and X_3 be independent random variables, each taking values 1 with probabilities 1/2. Define random variables Y_1, Y_2, and Y_3 by

$$Y_1=X_1X_2,\ Y_2=X_1X_3,\ Y_3=X_2X_3$$

Show that any two of these new random variables are independent but that Y_1, Y_2, and Y_3 are not independent. (6 marks)

b) Let $X' = (x_1, x_2, x_3)$ have multivariate normal density given by

$$f(x) = Cexp\left\{-\frac{1}{2}(3x_1^2 + x_2^2 + 5x_3^2 - x_1x_2 - 3x_2x_3)\right\}$$

Determine

i) Covariance matrix Σ (5 marks)

ii) Marginal density of x_1x_3 (5 marks)

Conditional density of x_1 given $x_2\,x_3$. (4 marks)

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