UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE) AND BACHELOR OF EDUCATION (ARTS)

SMA 320: METHODS 1

DATE: APRIL 11, 2018 **TIME: 11:00 AM – 1:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Define the following special functions: (5 marks)

   i) Laplace transform

   ii) Gamma functions

   iii) Beta functions

b) If $J_n(x)$ is Bessel's function of first kind of order n, prove that

$$J_{-n}(x) = (-1)^n J_n(x), \quad \text{for } n = 1,2,3,.......$$ (5 marks)

c) Let $\Gamma(x)$ denote the Gamma function. Prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$ (5 marks)

d) Find the Fourier sine series for $f(x) = x$ on $-L \le x \le L$ (5 marks)

e) If $B(x, y)$ denotes Beta function in two variables x and y, prove that

$$B(x+1, y) = \frac{x}{x+y} B(x, y)$$ (5 marks)

f) Show that the Laplace transform of $f(t)=t\sinh(t)$ is (5 marks)

$$\frac{2s}{\left(s^2-1\right)^2}$$

QUESTION TWO (20 MARKS)

a) Prove the given formula well known as Bonnet's recurrence formula for Legendre polynomial.

$(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$ (8 marks)

b) Use the Laplace differentiation theorem to evaluate the Laplace transformation of

(6 marks)

$$L\left\{t^2 \sin kt\right\}$$

c) Given that, $J_n(x)$ is Bessel's function of first kind of order n, prove that

$\frac{d}{dx}\left\{x^n J_n(x)\right\}=x^n J_{n-1}(x)$ for $n=1,2,3,.....$ (6 marks)

QUESTION THREE (20 MARKS)

a) If $\Gamma(x)$ and $B(x,y)$ denotes Gamma and Beta functions respectively, show that the two functions have the following relation

$B(x,y)=\frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ (8 marks)

b) Evaluate the given integral function

$\int_0^{\pi/2} \sin^8 \theta d\theta$ (7 marks)

c) Solve the partial differential equation using the method of separation of variables

$\frac{\partial^2 z}{\partial x \partial y}=x^2 y$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Find the Fourier cosine series for $f(x)=x^2$ on $-L \le x \le L$ (8 marks)

b) Use the convolution theorem to find the solution to the initial value differential equation

$x''+x=\cos t, \quad x(0)=0, \quad x'(0)=0$ (8 marks)

c) Determine a power series solution for the differential equation

$$y'' + y = 0$$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Show that any two different Legendre polynomials are orthogonal in the interval $-1 < x < 1$

(10 marks)

b) Solve the following initial value problem using Laplace transform. (10 marks)

$$y'' - 6y' + 10y = e^{-t}, \qquad y(0) = 0, \quad y'(0) = 1$$

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