UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE) AND BACHELOR OF EDUCATION (ARTS)

SMA 306: COMPLEX ANALYSIS I

DATE: APRIL 12, 2018 **TIME: 2:00 PM – 4:00 PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 MARKS)

a) Simplify $\frac{(\cos 4\theta+i\ \sin 4\theta)(\cos 3\varphi+i\sin 3\varphi)}{(\cos\theta-i\ \sin\theta)(\cos 2\varphi-i\sin 2\varphi)}$ (2 marks)

b) Express $z = -\sqrt{3} - i$ in polar form. (3 marks)

c) State the conditions necessary for a complex function to be continuous at point $z = z_0$ (2 marks)

d) State De Moivre's theorem and hence evaluate z^5 given $z = 1 + i\sqrt{3}$ (3 marks)

e) Show that $f(z) = e^z$ is a periodic function with period $2k\pi i$. (4 marks)

f) Show that $\cos^{-1} z = -i\ln(z + \sqrt{(z^2 - 1)})$ (4 marks)

g) Use Cauchy Integral formular to evaluate $\oint_c \frac{e^{2z}}{(z+1)^4}$ dz given C: $|z| = 3$ (3 marks)

h) Expand $\frac{1}{1+z}$ in a Taylor's series about point z = -2i. (4 marks)

i) Show that $\lim_{z\to i} \frac{z^2+1}{z^6+4} = \frac{1}{3}$ (2 marks)

j) Evaluate $(2 + i)^{1+i}$ (3 marks)

QUESTION TWO (20 MARKS)

a) Evaluate the following limits

$$\lim_{z\to 2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$$ (3 marks)

b) Find all the solutions of the equation $z^4 + 1 = 0$. (6 marks)

c) Evaluate $\int_{(1,3)}^{4,5} (2y + x^2)dx + (3x - y)dy$ along the straight line from $z_1 = 1 + 3i$ to $z_2 = 4 + 5i$. (7 marks)

d) Write the number $z = \sqrt{2}e^{i\frac{7\pi}{4}}$ in Cartesian form after sketching its position. (4 marks)

QUESTION THREE (20MARKS)

a) Show that the function $U(x, y) = e^{-y} \sin x$ is harmonic and hence find its harmonic conjugate V(x, y). (7 marks)

b) Evaluate $\frac{3 + i^5 - i^{11} + i^{13}}{i^4 + i^9 + i^{17}}$ (4 marks)

c) Use the Cauchy's integral to evaluate the integral $\oint_c \frac{z+1}{z^4+2iz^3}$ where C is circle such that |z| = 1. (6 marks)

d) Find the residue of $f(z) = \frac{1}{z^3 (1-z)^3}$ (3 marks)

QUESTION FOUR (20 MARKS)

a) Use the residue theorem to calculate $\int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}$ in a unit circle. (7 marks)

b) Find the image of the unit square in the z plane under transformation f(z) = Bz + C where B = 3 + 4i and C = 2 +3i. (4 marks)

c) Express $x^2 + y^2 = 25$ in terms of $z\bar{z}$ where z = x + iy. (4 marks)

d) Identify the singularities and hence compute the residues of the function $f(z) = \frac{1}{z^2(z^2+3z+2)}$ at each of its poles. (5 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate the contour integral $\oint_c z^2 dz$ along the path c from -1 +i to 5+3i compared of the line segments the first from -1 +i to 5+i and the second from 5 +i to 5+3i. (9 marks)

b) Find the Laurent series of the function $f(z) = \frac{e^{2z}}{(z-1)^3}$ about z = 1, hence name the singularity and the region of convergence. (6 marks)

c) Let F be analytic in simply connected domain D. Prove that if C is any closed curve in D then $\oint_C F(z)dz = 0$ (5 marks)

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