UNIVERSITY OF EMBU

2017/2018 ACADEMIC YEAR

SECOND SEMESTER EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 302: REAL ANALYSIS II

DATE: APRIL 11, 2018 TIME: 8:30 AM – 10:30 AM

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) Show that if a sequence of functions $\{f_n\}$ converges pointwise to a limit $f(x)$ on **K**, then this limit is unique (4 marks)

b) Let $f_n(x), g_n(x)$ be a sequence of functions of real numbers with limits $f(x), g(x)$ respectfully, and that $f_n(x) \leq g_n(x)\ \forall n \in N$, then prove that $f(x) < g(x)$. (5 marks)

c)

i) Define a function of bounded variation on a closed real valued interval $[a, b]$. (2 marks)

ii) Define a monotonic (increasing) function. Hence show that all monotonic functions on bounded interval are of bounded variation. (5 marks)

d) Show that if the function $f(x)$ is odd on the interval $(-\ell, \ell)$, then $\int_{-\ell}^{\ell} f(x)dx = 0$ (3 marks)

e) Define the Lebesgue Integral of the function f with respect to the measure μ over the set E. (3 marks)

f) Let $f(x) = x\ for\ a \leq x \leq b$ and define $\propto on\ [a,b]$ by $\propto x = 0$ for $a \leq x \leq b$ with

$\propto (b) = c$. If (P, t) is a tagged partition of $[a, b]$ with $P = \{x_0, x_1, x_2, \dots x_n\}$, define

$S(P, t, f, \propto) = t_n c$. Show that $\int_a^b x d \propto (x) = bc$ (5 marks)

g) Define the Fourier series of the function $f(x)$ on the interval $-l$ to l (3 marks)

QUESTION TWO: (20 MARKS)

a) Distinguish between pointwise convergence and uniform convergence of a sequence of functions $\{f_n\}$ on a space of real/complex valued space **K**. Hence for the sequence of functions $f_n : [0, 1] \to R$ defined by $f_n(x) = x^n$, if $0 \leq x < 1$, show that f_n is pointwise convergent but not uniformly convergent. (6 marks)

b) Prove that if f_n is a sequence of convergent functions of real numbers which converges uniformly to f and $f_n \neq 0$, $x \neq 0$ $\forall n \in N$, then $\frac{1}{f_n}$ is convergent to $\frac{1}{f}$ $(as\ n \to \infty)$

(7 marks)

c) State and prove the Weierstrauss M-test for convergence of real valued infinite series.

(7 marks)

QUESTION THREE (20 MARKS)

a) Define a Dirichlet function. Hence show that a Dirichlet function on the interval $[a, b]$ is not Riemann Integrable. (5 marks)

b) Briefly describe how the area under a given function f on the interval $[a, b]$ can be approximated using the Riemann-stieltjes Integration approach (4 marks)

c) Let $f(x) = 4x + 1$ for $0 \leq x \leq 1$ and $P = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$. Find The Riemann's' upper sum $U(P, f)$ and lower sum $L(P, f)$ of the function f defined on the interval $[a, b]$ (5 marks)

d) Let $f: [a, b] \to R$ be continuous and $f(a), f(b)$ have opposite signs, i.e. $f(a)f(b) < 0$, then there exists $c \in (a, b)$ such that $f(c) = 0$. (6 marks)

QUESTION FOUR (20 MARKS)

a) Prove that if f is a bounded variation function on $[a, b]$, then f is bounded (7 marks)

b) By partitioning method show that the Riemann Integral of the function $f(x) = 4x$ on the interval $0 \leq x \leq 1$ is 2 (7 marks)

c) By sketching graphs of the functions $f(x) = a^x$ and $f(x) = log_a x$ on the same axes ($taking\ a = 2$), identify any three differences in these functions. (6 marks)

QUESTION FIVE (20 MARKS)

a) Let f be a Lebesque integral $i.e\ f \in L(\mu)$on a collection of measurable sets E, prove that then $|f| \in L(\mu)$ on E and $|\int_E fd(\mu)| \leq \int_E |f|d(\mu)$. (5 marks)

b) Let $\sum_{n \in N} f_n$ be a series of real valued functions. Prove that if this series is absolutely convergent then it is convergent. By use of an appropriate example the converse of this is not necessarily true. (7 marks)

c) Find the Fourier series of the function defined by

$f(x) = x, \quad for -\pi < x < \pi$ (8 marks)

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