JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

COMPUTER SCIENCE, PHYSICAL SCIENCE, ANALYTICAL CHEMISTRY, INDUSTRIAL CHEMISTRY, CONTROL AND INSTRUMENTATION

CALCULUS II

TIME: 2hrs

Q1

a) Explain each of the following (2mks)
   - Implicit function
   - Parametric function

   Hence given that $x^2 - xy + y^2$=6 show that $\frac{d^2y}{dx^2} = \frac{36}{(x-2y)^3}$ (5mks)

b) Evaluate each of the following integrals
   - $\int \frac{x+1}{x^3+x^2-6x} dx$ (4mks)
   - $\int cos^8 sin^5 x dx$ (4mks)
   - $\int \frac{d\theta}{1+cos\theta} dx$ (4mks)

c) The rate of change of the voting population in Nyeri town with respect to time t in years is estimated by $N'^{(t)} = \frac{100t}{(1+t^2)^2}$ where N(t) is the voting population in thousands at any time t. if N(t) is 60,000 now determine the voting pattern 3years from now (3mks)

d) Show that $\frac{d}{dx}(sinh^{-1}3x) = \frac{1}{\sqrt{1+9x}}$ (3mks)

e) Given $z_1 = 4 - 7i \ \ and \ z_2 = -6 + 5i$ find $z_1\bar{z_2} - \bar{z_1}z_2$ hence write the polar form(4mks)

f) Find the surface area of a solid formed by revolving the region bounded by graphs

   $y = x^{\frac{1}{3}}$ where $1 \leq x \leq 2$ about the y axis (5mks)

Q2

a) Explain the following
   - Tangent line (1mk)
   - Normal line (1mk)

   Hence show that the curve whose parametric equation is $x = t + t^2$, $y = t^2$ then the value of $\frac{d^2y}{dx^2}$ for the curve when t=1 is $-\frac{1}{16}$ (3mks)

b) Find the arch length of the curve of $y = \frac{x^3}{6} + \frac{1}{2x}$ from x=2 to x=5 (4mks)

c) Use De Moivres Theorem to write $cos5\theta$ as a cosine function alone (4mks)

d) Evaluate
   - $\int e^{3x} sin4x dx$ (4mks)
   - $\int_2^5 x(3-x)^6\, dx$ (3mks)

Q4

a) Evaluate
   - $\int ln\, x dx$ (3mks)
   - $\int \frac{dx}{2+9x^2} dx$ (4mks)
   - $\int tan^5 x sec^6 x dx$ (4mks)

b) Find the tangent line to the curve $y^2 - 5y - x^2 = -4$ at $(-2,0)$ (4mks)

c) Discuss and sketch the curve $y = \frac{-2x^2}{x^2-4}$ (6mks)

Q4

a) Explain each of the following(2mks)
   - Proper rational function
   - Partial fraction

b) Express $\frac{x^2+4}{3x^3-4x^2-4x}$ in terms of fractions hence prove that

$\frac{x^2+4}{3x^3-4x^2-4x}dx = -ln|x| + \frac{1}{2}ln|x+2| + \frac{5}{6}ln|3x-2| + c$
(7mks)

c) Show that $sinh^{-1}x = ln\left|x + \sqrt{x^2+1}\right|$ (5mks)

d) If the rate of profit p'(t) in millions of Kenyan shillings from the sales of a new product in a city t years after it has been launched is given by $p'^{(t)} = 2t - te^{-t}$ find

- The profit accumulated during the first 8 years(4mks)
- The profit realized during the 5th year if p(0)=0 (2mks)

e) Evaluate $\int x^5 ln\, x dx$ (3mks)